Fractal Quantization of Speed in Physics of Numerical Relations

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The paper proposes a numeric-relational approach to the stability of real systems of coupled periodical processes and shows that it leads to fractal quantization of frequencies, wavelengths, and speeds caused by fractal scalar fields of transcendental numerical attractors. Applied to the stability of planetary systems, the approach predicts fractal quantization of orbits, orbital and rotational periods, and orbital speeds. On examples, the paper shows that the mean orbital speeds of planets, planetoids and large moons of the solar system are consistent with the prediction.

Introduction

Towards the end of the 19^{th} century, many physicists were convinced that the theoretical basics were complete and that there was nothing fundamentally new to discover. History proved them wrong. They didn't yet know quantum physics.

First it was the periodicity of the chemical properties of the elements, then there were regularities in the atomic spectra that pointed to a new physics.

Today, many physicists are convinced that the only thing that matters is to unite quantum theory with general relativity. But again, unexpected regularities appear on the empirical horizon, which are still dismissed as coincidences. This time we are dealing with regularities in the dynamics of planetary systems that cannot be derived from Kepler's laws or Einstein's theory of gravity. These are regularities in the distribution of orbital and rotational periods as well as gravitational parameters. Some of these regularities are highlighted in my papers [1, 2].

In the present article, we deal with regularities in the distribution of orbital speeds. For example, why is Jupiter's orbital speed identical to that of its moon Europa? Why is Saturn's orbital speed identical to that of its moon Dione? By the way, both moons are the fourth largest in their systems. Why is the orbital speed of Uranus identical to that of its moon Miranda? Why is the orbital speed of Jupiter's moon Io identical to that of the planetoid Ceres?

From the perspective of celestial mechanics, these regularities are not more than coincidences. From the perspective of our numeric-relational approach, these regularities are expected effects of a new relational physics.

Theoretical Approach

In a series of papers [1–6] and a book [7] I have introduced a numeric-relational approach to physics and demonstrated its application in particle physics, astrophysics, geophysics, engineering, and biophysics.

In particular, this approach leads to the conclusion that coupled periodical processes can avoid destabilizing mutual parametric resonance, if their frequency ratios approximate transcendental numbers. Among all transcendental numbers, Euler's number $e = 2.71828\ldots$ and Archimedes' number $\pi = 3.14159\ldots$ are unique. Indeed, the real power function of Euler's number is the only one that coincides with its own derivatives. In the consequence, Euler's number allows avoiding mutual parametric resonance between any coupled periodic processes including their derivatives [8]. In this way, Euler's number acts as primeval source of stability in systems of coupled periodic processes.

Archimedes' number determines the length of the circumference. The transcendence of the circumference avoids interruptions and makes impossible to define the start or endpoint of circular or elliptical motion. Hence, Archimedes' number makes possible eternal orbital motion, rotation, and oscillation. Perhaps that is why it is impossible to *completely* stop oscillations, for example, the thermal oscillations of atoms. In this way, Archimedes' number acts as primeval source of motion and kinetic energy.

Integer and rational powers of $e = 2.71828\ldots$ and $\pi = 3.14159\ldots$ form two complementary fractal scalar fields of transcendental attractors – the *Euler field* and the *Archimedes field*, as I have shown in [6]:

$$\mathcal{E} = e^{\mathcal{F}} \qquad \mathcal{A} = \pi^{\mathcal{F}}$$

Both fields are k-dimentional projections of the fundamental fractal $\mathcal{F}$ that is given by finite canonical continued fractions of integer attractors $n_0, n_1, n_2, \ldots, n_k$:

$$\mathcal{F} = \langle n_0; n_1, n_2, \ldots, n_k \rangle = n_0 + \cfrac{1}{n_1 + \cfrac{1}{n_2 + \cdots + \cfrac{1}{n_k}}}$$

In astronomical scales, the orbital periods and distances of the planets, planetoids and large moons in the solar system obey both the Euler field and the Archimedes field. Also their rotational periods obey the Euler field, as shown in [9]. In subatomic scales, the mass-ratios of elementary particles obey the Euler field [10].

Compared to the majority of known particles, electron and proton are exceptionally stable. Their life-spans top everything that is measurable, exceeding 10^{29} years [11]. This is why normal matter is formed by nucleons and electrons. According to our numeric-relational approach, electron and proton are stable, because the ratio of their eigenfrequencies approximates an integer power of Euler's number and its square root, which makes impossible proton-electron parametric resonance in their ground states.

The eigenfrequencies and harmonics of the proton and the electron are natural frequencies of any type of matter, also of the accreted matter of a planet. Given the enormous number of protons and electrons that form a planet, eigenresonance must be avoided in the long term. This affects any periodical process including orbital and rotational motion. This is why the planets in the solar system and in hundreds of exoplanetary systems have orbital periods that approximate integer and rational powers of Euler's number relative to the natural oscillation periods of the proton and the electron, as shown in my paper [1]. The perihelion and aphelion of a planetary orbit, if expressed in units of the Compton wavelength of the electron, give the lower and upper approximations of integer powers of Euler's number, as I have shown in [2]. As a consequence, the gravitational parameters of the Sun and its planets, if expressed in electron units, approximate integer powers of Euler's number. These findings allow us to interpret the approximation of integer powers of Euler's number and its roots as a general evolutional trend of numerical relations in real systems of many coupled periodical processes. This evolutionary trend drastically reduces the diversity of preferred orbital periods, distances, and speeds, increasing the likelihood of matches in different planetary or lunar systems.

Exemplary Applications

Since the orbital period of a planet approximates an integer power of Euler's number multiplied by the oscillation period of the electron, and its perihelion and aphelion approximate an integer power of Euler's number multiplied by the Compton wavelength of the electron, the orbital speed of the planet approximates the speed of light, divided by an integer power of Euler's number. For instance, Jupiter's distance from Sun approximates the 56^{th} power of Euler's number multiplied by the Compton wavelength of the electron $\lambda_e = 3.86159 \cdot 10^{-13}$ m. The aphelion 5.45492 AU = $8.160444 \cdot 10^{11}$ m delivers the upper approximation:

$$\ln\left(\frac{A(Jupiter)}{\lambda_e}\right) = \ln\left(\frac{8.160444 \cdot 10^{11}\,\text{m}}{3.86159 \cdot 10^{-13}\,\text{m}}\right) = 56.01$$

The perihelion 4.95029 AU = $7.405528 \cdot 10^{11}$ m delivers the lower approximation:

$$\ln\left(\frac{P(Jupiter)}{\lambda_e}\right) = \ln\left(\frac{7.405528 \cdot 10^{11}\,\text{m}}{3.86159 \cdot 10^{-13}\,\text{m}}\right) = 55.91$$

Jupiter's orbital period 4332.59 days equals the 66^{th} power of Euler's number multiplied by the oscillation period of the electron ($\tau_e = \lambda_e/c = 1.28809 \cdot 10^{-21}$ s is the angular oscillation period of the electron):

$$\ln\left(\frac{T(Jupiter)}{2\pi \cdot \tau_e}\right) = \ln\left(\frac{4332.59 \cdot 86400\,\text{s}}{2\pi \cdot 1.28809 \cdot 10^{-21}\,\text{s}}\right) = 66.00$$

Consequently, Jupiter's orbital speed approximates the speed of light, divided by the 10^{th} power of Euler's number, because 66 – 56 = 10. Indeed, Jupiter's average orbital speed equals 13.07 km/s:

$$\ln\left(\frac{V(Jupiter)}{c}\right) = \ln\left(\frac{13.07\,\text{km/s}}{c}\right) = -10.04$$

The orbital speed of Jupiter's fourth largest moon Europa approximates the same attractor $\mathcal{E}\langle -10\rangle$ of the Euler field. The average orbital speed of Europa equals 13.74 km/s:

$$\ln\left(\frac{V(Europa)}{c}\right) = \ln\left(\frac{13.74\,\text{km/s}}{c}\right) = -9.99$$

The orbital speeds of the other 3 Galilean moons of Jupiter approximate subattractors of the Euler field that correspond to reciprocal integer powers of Euler's number: The orbital speed of the moon Io approximates $\mathcal{E}\langle -10; +4\rangle$, the orbital speed of Ganimede approximates $\mathcal{E}\langle -10; -4\rangle$, and the orbital speed of Callisto approximates $\mathcal{E}\langle -10; -2\rangle$.

Venus' distance from Sun approximates the 54^{th} power of Euler's number multiplied by the Compton wavelength of the electron λ_e. The aphelion 0.728213 AU = $1.08939 \cdot 10^{11}$ m delivers the upper approximation:

$$\ln\left(\frac{A(Venus)}{\lambda_e}\right) = \ln\left(\frac{1.08939 \cdot 10^{11}\,\text{m}}{3.86159 \cdot 10^{-13}\,\text{m}}\right) = 54.00$$

The perihelion 0.718440 AU = $1.07477 \cdot 10^{11}$ m delivers the lower approximation:

$$\ln\left(\frac{P(Venus)}{\lambda_e}\right) = \ln\left(\frac{1.07477 \cdot 10^{11}\,\text{m}}{3.86159 \cdot 10^{-13}\,\text{m}}\right) = 53.98$$

The orbital period 224.701 days of Venus approximates the 63^{th} power of Euler's number multiplied by the oscillation period of the electron:

$$\ln\left(\frac{T(Venus)}{2\pi \cdot \tau_e}\right) = \ln\left(\frac{224.701 \cdot 86400\,\text{s}}{2\pi \cdot 1.28809 \cdot 10^{-21}\,\text{s}}\right) = 63.04$$

Consequently, the orbital speed of Venus approximates the speed of light, divided by the 9^{th} power of Euler's number, because 63 – 54 = 9. In fact, the average orbital speed of Venus equals 35.02 km/s:

$$\ln\left(\frac{V(Venus)}{c}\right) = \ln\left(\frac{35.02\,\text{km/s}}{c}\right) = -9.05$$

Since e and π are transcendental, there are no rational powers of these numbers that can produce identical results. Therefore, attractors of the Archimedes field are different from attractors of the Euler field. For instance, the mean orbital speed 9.68 km/s of Saturn does not approximate a main attractor of the Euler field, but approximates the main attractor $\mathcal{A}\langle -9 \rangle$ of the Archimedes field:

$$\mathrm{lp}\left(\frac{V(Saturn)}{c}\right) = \mathrm{lp}\left(\frac{9.68 \text{ km/s}}{c}\right) = -9.03$$

We use the symbol "lp" for the logarithm to the base π:

$$\mathrm{lp}(x) = \frac{\ln(x)}{\ln(\pi)}$$

This circumstance suggests that transcendental relations not only stabilize orbits preventing them from mutual parametric resonance, but also assign orbits to the numerical field to which they belong.

For instance, the mean orbital speed 4.743 km/s of Pluto approximates the main attractor $\mathcal{E}\langle -11 \rangle$ of the Euler field:

$$\ln\left(\frac{V(Pluto)}{c}\right) = \ln\left(\frac{4.743 \text{ km/s}}{c}\right) = -11.05$$

while the mean orbital speed 3.434 km/s of Eris approximates the main attractor $\mathcal{A}\langle -10 \rangle$ of the Archimedes field:

$$\mathrm{lp}\left(\frac{V(Eris)}{c}\right) = \mathrm{lp}\left(\frac{3.434 \text{ km/s}}{c}\right) = -9.94$$

Also the mean orbital speed 29.7827 km/s of the Earth does not approximate a main attractor of the Euler field, but approximates the main attractor $\mathcal{A}\langle -8 \rangle$ of the Archimedes field:

$$\mathrm{lp}\left(\frac{V(Earth)}{c}\right) = \mathrm{lp}\left(\frac{29.7827 \text{ km/s}}{c}\right) = -8.05$$

Possibly this indicates a transcendental duality of Euler- and Archimedes-orbits in the solar system. The orbital speeds $V_E(\mathcal{F})$ belong to the Euler field while the $V_A(\mathcal{F})$ belong to the Archimedes field:

$$V_E(\mathcal{F}) = \frac{c}{e^{\mathcal{F}}} \qquad V_A(\mathcal{F}) = \frac{c}{\pi^{\mathcal{F}}}$$

Orbital speeds that correspond to the base layer of the fundamental fractal $\mathcal{F}$ approximate the speed of light divided by integer powers of e and π.

Conclusion

In [6] I have shown that the proposed here numeric-relational approach to the stability of real systems of coupled periodical processes predicts a fractal quantization of frequencies and wavelengths caused by fractal scalar fields of transcendental numerical attractors – the Euler field and the Archimedes field. In the case of planetary systems, for example the solar system, the Euler field and the Archimedes field cause a fractal quantization of orbits and orbital periods.

The current article shows that stable orbital speeds derive from the speed of light divided by integer and reciprocal integer powers of e or π. This circumstance drastically reduces the diversity of preferred orbits, orbital periods, and speeds, increasing the likelihood of matches in different planetary or lunar systems. This is why in the solar system, the orbital speeds of some moons coincide with the orbital speeds of some planets and planetoids. Considering the described here fractal quantization of orbits as general evolutional trend, orbital speeds corresponding to integer powers of e or π should be widespread in the galaxy.

The duality of Euler- and Archimedes-orbits in the solar system suggests that the orbits differ in their function. Based on my previous research [6], I hypothesize that Euler-orbits act as stabilizers of the system, while Archimedes-orbits act as energizers.

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