Fractal Scaling Models of Natural Oscillations in Chain Systems and the Mass Distribution of Particles

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The paper presents a fractal scaling model of a chain system of quantum harmonic oscillators, that reproduces some systematic features in the mass distribution of hadrons, leptons and gauge bosons.

1 Introduction

The origin of particle masses is one of the most important unsolved problems of modern physics. Also the discrete character of the distribution of particle masses is untreated. In this paper we won't discuss the current situation in the standard theory. Based on a fractal scaling model [1] of natural oscillations in chain systems of harmonic quantum oscillators we will analyze the distributions of particles in dependence on their masses to find out systematic features.

Fractal scaling models [2] of natural oscillations are not based on any statements about the nature of the link or interaction between the elements of the oscillating chain system. Therefore the model statements are quite generally, what opens a wide field of possible applications. Logarithmic scaling is a well known property of inclusive distributions in high energy particle reactions [3]. The quantity of secondary particles increases in dependence on the logarithm of the collision energy.

In the framework of the standard theory, the electron is stable because it's the least massive particle with non-zero electric charge. Its decay would violate charge conservation. The proton is stable, because it's the lightest baryon and the baryon number is conserved. Therefore the proton is the most important baryon, while the electron is the most important lepton and the proton-to-electron mass ratio can be understood as a fundamental physical constant. In the framework of the standard theory, the W- and Z-bosons are elementary particles that mediate the weak force. The rest masses of all theses particles are measured with high precision. The masses of other elementary or stable particles (quarks, neutrinos) are unknown.

In the framework of our model [1], particles are resonance states in chain systems of harmonic quantum oscillators and the masses of fundamental particles are connected by the scaling exponent $\frac{3}{2}$. For example, the proton-to-electron mass ratio is $7\frac{1}{2}$, but the W-boson-to-proton mass ratio is $4\frac{1}{2}$. This means, they are connected by the equation:

$$\ln\left(\frac{m_w}{m_p}\right) = \ln\left(\frac{m_p}{m_e}\right) - 3\,. \tag{1}$$

Therefore the W-boson-to-electron mass ratio corresponds to $4\frac{1}{2} + 7\frac{1}{2} = 12$:

$$\ln\left(\frac{m_w}{m_e}\right) = 12. \tag{2}$$

Already within the eighties the scaling exponent $\frac{3}{2}$ was found in the distribution of particle masses by Valery A. Kolombet [4]. In addition, we have shown [2] that the masses of the most massive celestial bodies in the Solar System are connected by the same scaling exponent $\frac{3}{2}$. The scaling exponent $\frac{3}{2}$ arises as consequence of natural oscillations in chain systems of similar harmonic oscillators [1]. If the natural frequency of one harmonic oscillator is known, one can calculate the complete fractal spectrum of natural frequencies of the chain system, in which spectral nodes arise on the distance of 1 and $\frac{1}{2}$ logarithmic units.

Near spectral nodes the spectral density reaches local maximum and natural frequencies are distributed maximum densely. The energy efficiency of natural oscillations is very high. Therefore one can expect that spectral nodes represent states of the oscillating chain system, which have the highest degree of effectiveness. For this reason we suspect, that stable particles correspond to main spectral nodes.

2 Methods

Based on the continued fraction method [5] we will search the natural frequencies of a chain system of many similar harmonic oscillators in this form:

$$f_{jk} = f_{00} \exp\left(S_{jk}\right). \tag{3}$$

f_{jk} is a set of natural frequencies of a chain system of similar harmonic oscillators, f_{00} is the natural oscillation frequency of one oscillator, S_{jk} is a set of finite continued fractions with integer elements:

$$S_{jk} = n_{j0} + \cfrac{1}{n_{j1} + \cfrac{1}{n_{j2} + \ddots + \cfrac{1}{n_{jk}}}} = [n_{j0}; n_{j1}, n_{j2}, \ldots, n_{jk}]\,, \tag{4}$$

where $n_{j0}, n_{j1}, n_{j2}, \ldots, n_{jk} \in Z$, $j = 0\,, \infty$. We investigate continued fractions (4) with a finite quantity of layers k, which generate discrete spectra, because in this case all S_{jk} represent rational numbers. Therefore the free links n_{j0} and the partial denominators $n_{j1}, n_{j2}, \ldots, n_{jk}$ can be interpreted as "quantum

Particle	Rest mass m, MeV/c^2 [6]	$\ln(m/m_{00})$	S	d
electron (m_{00})	0.510998910 ± 0.000000013	0	[0]	0.000
proton	938.27203 ± 0.00008	7.515	[7; 2]	0.015
neutron	939.565346 ± 0.000023	7.517	[7; 2]	0.017
W-boson	80398 ± 25	11, 966	[12]	−0.034
Z-boson	91187.6 ± 2.1	12.092	[12]	0,092

Table 1: The rest masses of well measured stable and fundamental particles and the S-values (4) of the nearest main spectral nodes for the electron calibrated model spectrum. The deviation $d = (\ln(m/m_{00}) - S)$ is indicated.

numbers". The present paper follows the Terskich [5] definition of a chain system, where the interaction between the elements proceeds only in their movement direction. Model spectra (4) are not only logarithmic-invariant, but also fractal, because the discrete hyperbolic distribution of natural frequencies f_{jk} repeats itself on each spectral layer. The partial denominators run through positive and negative integer values. Ranges of relative low spectral density (spectral gaps) and ranges of relative high spectral density (spectral nodes) arise on each spectral layer. In addition to the first spectral layer, Fig. 1 shows the second spectral layer $k = 2$ with $|n_{j1}| = 2$ (logarithmic representation). Maximum spectral density areas (spectral nodes) arise automatically on the distance of integer and half logarithmic units.

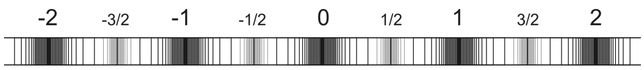

Fig. 1: The spectrum (4) on the first layer $k = 1$, for $|n_{j0}| = 0,\ 1\ 2, \ldots$ and $|n_{j1}| = 2,\ 3,\ 4, \ldots$ and, in addition, the second spectral layer $k = 2$, with $|n_{j1}| = 2$ and $|n_{j2}| = 2,\ 3,\ 4,\ \ldots$ (logarithmic representation).

Fractal scaling models of natural oscillations are not based on any statements about the nature of the link or interaction between the elements of the oscillating chain system. For this reason we assume that our model could be useful also for the analysis of natural oscillations in chain systems of harmonic quantum oscillators. We assume that in the case of natural oscillations the amplitudes are low, the oscillations are harmonic and the oscillation energy E depends only on the frequency (h is the Planck constant):

$$E = hf\,. \tag{5}$$

In the framework of our model (3) all particles are resonances, in which to the oscillation energy (5) corresponds the particle mass m:

$$m = f\frac{h}{c^2}\,. \tag{6}$$

In this connection the equation (6) means that quantum oscillations generate mass. Under consideration of (3) now we can create a fractal scaling model of the mass spectrum of model particles. This mass spectrum is described by the same continued fraction (4), for $m_{00} = f_{00}\frac{h}{c^2}$:

$$\ln\left(\frac{m_{jk}}{m_{00}}\right) = [n_{j0}; n_{j1}, n_{j2}, \ldots, n_{jk}]\,. \tag{7}$$

The frequency spectrum (4) and the mass spectrum (7) are isomorphic. The mass spectrum (7) is fractal and consequently it has a clear hierarchical structure, in which continued fractions (4) of the form $[n_{j0}]$ and $[n_{j0}; 2]$ define main spectral nodes, as Fig. 1 shows.

3 Results

In the present paper we will compare the scaling model mass spectrum (7) in the range of 100 KeV/c^2 to 100 GeV/c^2 with the mass distribution of well-known particles — hadrons, leptons and gauge bosons.

The model mass spectrum (7) is logarithmically symmetrical and the main spectral nodes arise on the distance of 1 and $\frac{1}{2}$ logarithmic units, as fig. 1 shows. The mass m_{00} in (7) corresponds to the main spectral node $S_{00} = [0]$, because $\ln(m_{00}/m_{00}) = 0$. Let's assume that m_{00} is the electron rest mass 0.510998910(13) MeV/c^2 [6]. In this case (7) describes the mass spectrum that corresponds to the natural frequency spectrum (4) of a chain system of vibrating electrons. Further stable or fundamental model particles correspond to further main spectral nodes of the form $[n_{j0}]$ and $[n_{j0}; 2]$. Actually, near the node [12] we find the W- and Z-bosons, but near the node [7; 2] the proton and neutron masses, as Table 1 shows.

Theoretically, a chain system of vibrating protons generates the same spectrum (7). Also in this case, stable or fundamental model particles correspond to main spectral nodes of the form $[n_{j0}]$ and $[n_{j0}; 2]$, but relative to the electron calibrated spectrum, they are moved by $-7\frac{1}{2}$ logarithmic units. Actually, if m_{00} is the proton rest mass 938.27203(8) MeV/c^2 [6], then the electron corresponds to the node $[-7; -2]$, but the W- and Z-bosons correspond to node [4; 2].

Consequently, the core claims of our model don't depend on the selection of the calibration mass m_{00}, if it is the rest mass of a fundamental resonance state that corresponds to a main spectral node. As mentioned already, this is why the model spectrum (7) is logarithmically symmetrical.

Because a chain system of any similar harmonic oscillators generates the spectrum (7), m_{00} can be much more smaller than the electron mass. Only one condition has to be fulfilled: m_{00} has to correspond to a main spectral node of the model spectrum (7). On this background all particles

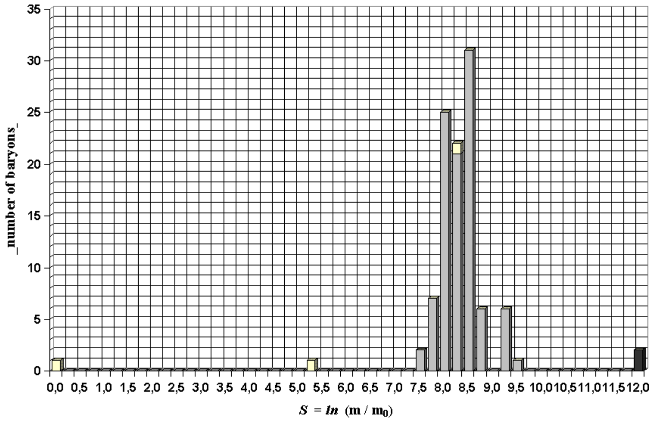

Fig. 2: This histogram was built based on Table 2 and shows the distribution of baryons (grey bars) and leptons (white bars) over $\frac{1}{4}$ logarithmic units wide S-intervals in the range of the electron mass ($S = 0$, white bar) to the W- and Z-bosons ($S = 12$, black bar).

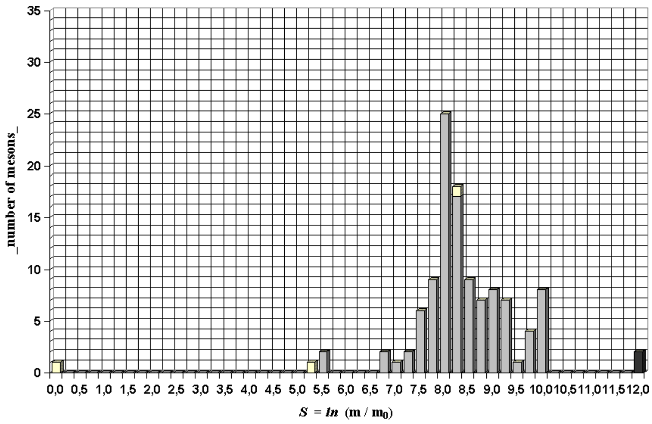

Fig. 3: This histogram was built based on Table 3 and shows the distribution of mesons (grey bars) and leptons (white bars) over $\frac{1}{4}$ logarithmic units wide S-intervals in the range of the electron mass ($S = 0$, white bar) to the W- and Z-bosons ($S = 12$, black bar).

can be interpreted as resonance states in a chain system of harmonic quantum oscillators, in which the rest mass of each single oscillator goes to zero. In the framework of our oscillation model this way can be understood the transition of massless to massive states.

In our model massive particles don't arise because of a symmetry violation. Massive particles arise as resonance states and their mass distribution is logarithmically symmetric.

Further we will investigate the distribution of hadrons (baryons and mesons) in dependence on their rest masses. For this we will split up the mass spectrum (7) into equal in size logarithmic intervals and build histograms. To separate clear the main spectral nodes $[n_{j0}]$ and $[n_{j0}; 2]$, we have to split up the spectrum (7) into S-intervals of $\frac{1}{4}$ logarithmic units.

Table 2 shows the measured masses of baryons, the calculated S-intervals of $\frac{1}{4}$ logarithmic units width and the corresponding calculated mass-intervals. Based on Table 2 a histogram was built (Fig. 2) that shows the distribution of baryons over the $\frac{1}{4}$ logarithmic S-intervals. Based on Table 3,

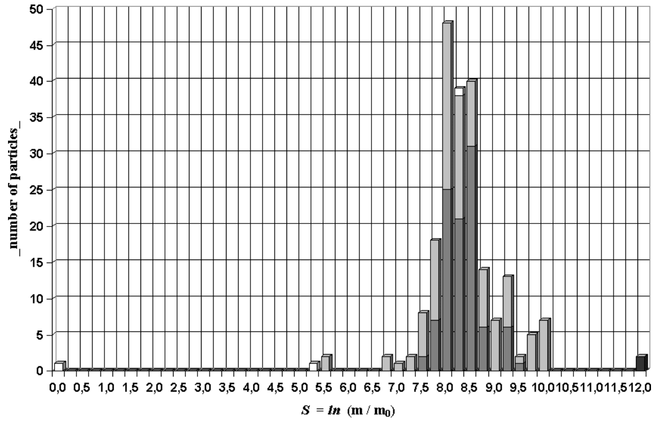

Fig. 4: This histogram was built based on tables 2, 3, 4, 5 and shows the distribution of baryons (dark grey bars), mesons (light grey bars) and leptons (white bars) over $\frac{1}{4}$ logarithmic units wide S-intervals in the range of the electron mass ($S = 0$, white bar) to the W- and Z-bosons ($S = 12$, black bar).

Figure 3 shows the distribution of mesons, but Figure 4 shows the distribution of baryons, mesons, leptons and gauge bosons over the $\frac{1}{4}$ logarithmic S-intervals in the range of 0 to 12 logarithmic units.

All known baryons are distributed over an interval of 2 logarithmic units, of $S = [7; 2]$ to $S = [9; 2]$, as Figure 2 shows. Maximum of baryons occupy the logarithmic center $S = [8; 2]$ of this interval. Figure 3 shows that maximum of mesons occupy the spectral node $S = [8]$ that split up the interval of $S = [0]$ to $S = [12]$ between the electron and the W- and Z-bosons proportionally of $\frac{2}{3}$.

The mass distribution of leptons isn't different of the baryon and meson mass distributions, but follows them, as Figure 4 shows. The mass of the most massive lepton (tauon) is near the maximum of the baryon and meson mass distributions, as Figures 2–4 show.

4 Resume

In the framework of the present model discrete scaling mass distributions arise as result of natural oscillations in chain systems of harmonic quantum oscillators. The observable mass distributions of baryons, mesons, leptons and gauge bosons are connected by the model scaling exponent $\frac{2}{3}$. In addition, with high precision, the masses of known fundamental and stable particles are connected by the model scaling exponent $\frac{3}{2}$. Presumably, the complete mass distribution of particles is logarithmically symmetric and, possibly, massive particles don't arise because of a symmetry violation, but as resonance states in chain systems of quantum oscillators.

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Table 2. The measured masses of baryons [6], the calculated S-intervals of $\frac{1}{4}$ logarithmic units width and the corresponding calculated mass-intervals.

baryons	measured mass MeV/c^2	mass interval MeV/c^2	S-interval	S
N-baryons, $S = 0$, $I = 1/2$				
proton	938.27203 ± 0.00008	815 – 1047	7.375 – 7.625	[7; 2]
neutron	939.565346 ± 0.000023	815 – 1047	7.375 – 7.625	[7; 2]
N(1440)	1420 – 1470	1344 – 1726	7.875 – 8.125	[8]
N(1520)	1515 – 1525	1344 – 1726	7.875 – 8.125	[8]
N(1650)	1645 – 1670	1344 – 1726	7.875 – 8.125	[8]
N(1675)	1670 – 1680	1344 – 1726	7.875 – 8.125	[8]
N(1680)	1680 – 1690	1344 – 1726	7.875 – 8.125	[8]
N(1710)	1680 – 1740	1344 – 1726	7.875 – 8.125	[8]
N(1720)	1700 – 1750	1344 – 1726	7.875 – 8.125	[8]
N(2190)	2100 – 2200	1726 – 2216	8.125 – 8.375	[8; 4]
N(2220)	2200 – 2300	2216 – 2846	8.375 – 8.625	[8; 2]
N(2250)	2200 – 2350	2216 – 2846	8.375 – 8.625	[8; 2]
N(2600)	2550 – 2750	2216 – 2846	8.375 – 8.625	[8; 2]
Δ-baryons, $S = 0$, $I = 1/2$				
Δ(1232)	1231 – 1233	1047 – 1344	7.625 – 7.875	[8; −4]
Δ(1600)	1550 – 1700	1344 – 1726	7.875 – 8.125	[8]
Δ(1620)	1600 – 1660	1344 – 1726	7.875 – 8.125	[8]
Δ(1700)	1670 – 1750	1344 – 1726	7.875 – 8.125	[8]
Δ(1905)	1865 – 1915	1726 – 2216	8.125 – 8.375	[8; 4]
Δ(1910)	1870 – 1920	1726 – 2216	8.125 – 8.375	[8; 4]
Δ(1920)	1900 – 1970	1726 – 2216	8.125 – 8.375	[8; 4]
Δ(1930)	1900 – 2020	1726 – 2216	8.125 – 8.375	[8; 4]
Δ(1950)	1915 – 1950	1726 – 2216	8.125 – 8.375	[8; 4]
Δ(2420)	2300 – 2500	2216 – 2846	8.375 – 8.625	[8; 2]
Λ-baryons, $S = -1$, $I = 0$				
Λ	1115.683 ± 0.006	1047 – 1344	7.625 – 7.875	[8; −4]
Λ(1405)	1406 ± 4	1344 – 1726	7.875 – 8.125	[8]
Λ(1520)	1519.5 ± 1.0	1344 – 1726	7.875 – 8.125	[8]
Λ(1600)	1560 – 1700	1344 – 1726	7.875 – 8.125	[8]
Λ(1670)	1660 – 1680	1344 – 1726	7.875 – 8.125	[8]
Λ(1690)	1685 – 1695	1344 – 1726	7.875 – 8.125	[8]
Λ(1800)	1720 – 1850	1726 – 2216	8.125 – 8.375	[8; 4]
Λ(1810)	1750 – 1850	1726 – 2216	8.125 – 8.375	[8; 4]
Λ(1820)	1815 – 1825	1726 – 2216	8.125 – 8.375	[8; 4]
Λ(1830)	1810 – 1830	1726 – 2216	8.125 – 8.375	[8; 4]
Λ(1890)	1850 – 1910	1726 – 2216	8.125 – 8.375	[8; 4]
Λ(2100)	2090 – 2110	1726 – 2216	8.125 – 8.375	[8; 4]
Λ(2110)	2090 – 2140	1726 – 2216	8.125 – 8.375	[8; 4]
Λ(2350)	2340 – 2370	2216 – 2846	8.375 – 8.625	[8; 2]
Σ-baryons, $S = -1$, $I = 1$				
Σ^+	1189.37 ± 0.07	1047 – 1344	7.625 – 7.875	[8; −4]
Σ^0	1192.642 ± 0.024	1047 – 1344	7.625 – 7.875	[8; −4]
Σ^-	1197.449 ± 0.030	1047 – 1344	7.625 – 7.875	[8; −4]
$\Sigma(1385)^+$	1382.8 ± 0.4	1344 – 1726	7.875 – 8.125	[8]
$\Sigma(1385)^0$	1383.7 ± 1.0	1344 – 1726	7.875 – 8.125	[8]
$\Sigma(1385)^-$	1387.2 ± 0.5	1344 – 1726	7.875 – 8.125	[8]
Σ(1660)	1630 – 1690	1344 – 1726	7.875 – 8.125	[8]
Σ(1670)	1665 – 1685	1344 – 1726	7.875 – 8.125	[8]
Σ(1750)	1730 – 1800	1726 – 2216	8.125 – 8.375	[8; 4]
Σ(1775)	1770 – 1780	1726 – 2216	8.125 – 8.375	[8; 4]
Σ(1915)	1900 – 1935	1726 – 2216	8.125 – 8.375	[8; 4]
Σ(1940)	1900 – 1950	1726 – 2216	8.125 – 8.375	[8; 4]
Σ(2030)	2025 – 2040	1726 – 2216	8.125 – 8.375	[8; 4]
Σ(2250)	2210 – 2280	2216 – 2846	8.375 – 8.625	[8; 2]
Ξ-baryons, $S = -2$, $I = 1/2$				
Ξ^0	1314.86 ± 0.20	1047 – 1344	7.625 – 7.875	[8; −4]
Ξ^-	1321.71 ± 0.07	1047 – 1344	7.625 – 7.875	[8; −4]
$\Xi(1530)^0$	1531.80 ± 0.32	1344 – 1726	7.875 – 8.125	[8]
$\Xi(1530)^-$	1535.0 ± 0.6	1344 – 1726	7.875 – 8.125	[8]
Ξ(1690)	1690 ± 10	1344 – 1726	7.875 – 8.125	[8]
Ξ(1820)	1823 ± 5	1726 – 2216	8.125 – 8.375	[8; 4]
Ξ(1950)	1950 ± 15	1726 – 2216	8.125 – 8.375	[8; 4]
Ξ(2030)	2025 ± 5	1726 – 2216	8.125 – 8.375	[8; 4]

baryons	measured mass MeV/c^2	mass interval MeV/c^2	S-interval	S
Ω-baryons, $S = -3$, $I = 0$				
Ω^-	1672.45 ± 0.29	1344 – 1726	7.875 – 8.125	[8]
$\Omega(2250)^-$	2252 ± 9	2216 – 2846	8.375 – 8.625	[8; 2]
charmed baryons, $C = +1$				
Λ_c^+	2286.46 ± 0.14	2216 – 2846	7.375 – 8.625	[8; 2]
$\Lambda_c(2595)^+$	2595.4 ± 0.6	2216 – 2846	7.375 – 8.625	[8; 2]
$\Lambda_c(2625)^+$	2628.1 ± 0.6	2216 – 2846	7.375 – 8.625	[8; 2]
$\Lambda_c(2880)^+$	2881.53 ± 0.35	2846 – 3654	8.625 – 8.875	[9; −4]
$\Lambda_c(2940)^+$	2939.3 ± 1.5	2846 – 3654	8.625 – 8.875	[9; −4]
$\Sigma_c(2455)^{++}$	2454.02 ± 0.18	2216 – 2846	8.375 – 8.625	[8; 2]
$\Sigma_c(2455)^+$	2452.9 ± 0.4	2216 – 2846	8.375 – 8.625	[8; 2]
$\Sigma_c(2455)^0$	2453.76 ± 0.18	2216 – 2846	8.375 – 8.625	[8; 2]
$\Sigma_c(2801)^{++}$	2801 ± 6	2216 – 2846	8.375 – 8.625	[8; 2]
$\Sigma_c(2800)^+$	2792 ± 14	2216 – 2846	8.375 – 8.625	[8; 2]
$\Sigma_c(2800)^0$	2802 ± 7	2216 – 2846	8.375 – 8.625	[8; 2]
Ξ_c^+	2467.8 ± 0.6	2216 – 2846	8.375 – 8.625	[8; 2]
Ξ_c^0	2470.88 ± 0.8	2216 – 2846	8.375 – 8.625	[8; 2]
$\Xi_c'^+$	2575.6 ± 3.1	2216 – 2846	8.375 – 8.625	[8; 2]
$\Xi_c'^0$	2577.9 ± 2.9	2216 – 2846	8.375 – 8.625	[8; 2]
$\Xi_c(2645)^+$	2645.9 ± 0.6	2216 – 2846	8.375 – 8.625	[8; 2]
$\Xi_c(2645)^0$	2645.9 ± 0.5	2216 – 2846	8.375 – 8.625	[8; 2]
$\Xi_c(2790)^+$	2789.1 ± 3.2	2216 – 2846	8.375 – 8.625	[8; 2]
$\Xi_c(2790)^0$	2791.8 ± 3.3	2216 – 2846	8.375 – 8.625	[8; 2]
$\Xi_c(2815)^+$	2816.6 ± 0.9	2216 – 2846	8.375 – 8.625	[8; 2]
$\Xi_c(2815)^0$	2819.6 ± 1.2	2216 – 2846	8.375 – 8.625	[8; 2]
$\Xi_c(2980)^+$	2971.4 ± 3.3	2846 – 3654	8.625 – 8.875	[9; −4]
$\Xi_c(2880)^0$	2968.0 ± 2.6	2846 – 3654	8.625 – 8.875	[9; −4]
$\Xi_c(3080)^+$	3077.0 ± 0.4	2846 – 3654	8.625 – 8.875	[9; −4]
$\Xi_c(3080)^0$	3079.9 ± 1.4	2846 – 3654	8.625 – 8.875	[9; −4]
Ω_c^0	2695.2 ± 1.7	2216 – 2846	8.375 – 8.625	[8; 2]
$\Omega_c(2770)^0$	2765.9 ± 2.0	2216 – 2846	8.375 – 8.625	[8; 2]
bottom baryons, $B = -1$				
Λ_b^0	5620.2 ± 1.6	4692 – 6025	9.125 – 9.375	[9; 4]
Σ_b^+	5807.8 ± 2.7	4692 – 6025	9.125 – 9.375	[9; 4]
Σ_b^-	5815.2 ± 2.0	4692 – 6025	9.125 – 9.375	[9; 4]
Σ_b^{*+}	5829.0 ± 3.4	4692 – 6025	9.125 – 9.375	[9; 4]
Σ_b^{*-}	5836.4 ± 2.8	4692 – 6025	9.125 – 9.375	[9; 4]
Ξ_b	5792.4 ± 3.0	4692 – 6025	9.125 – 9.375	[9; 4]
Σ_b^-	6165 ± 16	6025 – 7736	9.375 – 9.625	[9; 2]

Table 3. The measured masses of mesons [6], the calculated S-intervals of $\frac{1}{4}$ logarithmic units width and the corresponding calculated mass-intervals.

mesons	measured mass MeV/c^2	mass interval MeV/c^2	S-interval	S
light unflavored mesons $S = C = B = 0$				
$\pi^\pm$	139.57018 ± 0.00035	110 – 142	5.375 – 5.625	[5; 2]
π^0	134.9766±0.0006	110 – 142	5.375 – 5.625	[5; 2]
η	547.853 ± 0.024	495 – 635	6.875 – 7.125	[7]
$\rho(770)$	775.49 ± 0.34	635 – 815	7.125 – 7.375	[7; 4]
$\omega(782)$	782.65 ± 0.12	635 – 815	7.125 – 7.375	[7; 4]
$\rho'(958)$	957.78 ± 0.06	815 – 1047	7.375 – 7.626	[7; 2]
$f_0(980)$	980 ± 10	815 – 1047	7.375 – 7.626	[7; 2]
$a_0(980)$	980 ± 20	815 – 1047	7.375 – 7.626	[7; 2]

mesons	measured mass MeV/c^2	mass interval MeV/c^2	S-interval	S
$\phi(1020)$	1019.455 ± 0.020	815 – 1047	7.375 – 7.626	[7; 2]
$a_0(980)$	980 ± 20	815 – 1047	7.375 – 7.626	[7; 2]
$\phi(1020)$	1019.455 ± 0.020	815 – 1047	7.375 – 7.626	[7; 2]
$h_1(1170)$	1170 ± 20	1047 – 1344	7.626 – 7.875	[8; −4]
$b_1(1235)$	1229.5 ± 3.2	1047 – 1344	7.626 – 7.875	[8; −4]
$a_1(1260)$	1230 ± 40	1047 – 1344	7.626 – 7.875	[8; −4]
$f_2(1270)$	1275.1 ± 1.2	1047 – 1344	7.626 – 7.875	[8; −4]
$f_1(1285)$	1281.8 ± 0.6	1047 – 1344	7.626 – 7.875	[8; −4]
$\eta(1295)$	1294 ± 4	1047 – 1344	7.626 – 7.875	[8; −4]
$h_1(1170)$	1170 ± 20	1047 – 1344	7.626 – 7.875	[8; −4]
$b_1(1235)$	1229.5 ± 3.2	1047 – 1344	7.626 – 7.875	[8; −4]
$a_1(1260)$	1230 ± 40	1047 – 1344	7.626 – 7.875	[8; −4]
$f_2(1270)$	1275.1 ± 1.2	1047 – 1344	7.626 – 7.875	[8; −4]
$f_1(1285)$	1281.8 ± 0.6	1047 – 1344	7.626 – 7.875	[8; −4]
$\eta(1295)$	1294 ± 4	1047 – 1344	7.626 – 7.875	[8; −4]
$\pi(1300)$	1300 ± 100	1047 – 1344	7.626 – 7.875	[8; −4]
$a_2(1320)$	1318.3 ± 0.6	1047 – 1344	7.626 – 7.875	[8; −4]
$f_0(1370)$	1200 – 1500	1344 – 1726	7.875 – 8.125	[8]
$\pi_1(1400)$	1351 ± 30	1344 – 1726	7.875 – 8.125	[8]
$\eta(1450)$	1409.8 ± 2.5	1344 – 1726	7.875 – 8.125	[8]
$f_1(1420)$	1426.4 ± 0.9	1344 – 1726	7.875 – 8.125	[8]
$\omega(1400)$	1400 – 1450	1344 – 1726	7.875 – 8.125	[8]
$a0(1450)$	1474 ± 19	1344 – 1726	7.875 – 8.125	[8]
$\rho(1450)$	1465 ± 25	1344 – 1726	7.875 – 8.125	[8]
$\eta(1475)$	1476 ± 4	1344 – 1726	7.875 – 8.125	[8]
$f_0(1500)$	1505 ± 6	1344 – 1726	7.875 – 8.125	[8]
$f_2(1525)$	1525 ± 5	1344 – 1726	7.875 – 8.125	[8]
$\pi_1(1600)$	1662 ± 15	1344 – 1726	7.875 – 8.125	[8]
$\eta_2(1645)$	1617 ± 5	1344 – 1726	7.875 – 8.125	[8]
$\omega(1650)$	1670 ± 30	1344 – 1726	7.875 – 8.125	[8]
$\omega_3(1670)$	1667 ± 4	1344 – 1726	7.875 – 8.125	[8]
$\pi_2(1670)$	1672.4 ± 3.2	1344 – 1726	7.875 – 8.125	[8]
$\phi(1680)$	1680 ± 20	1344 – 1726	7.875 – 8.125	[8]
$\rho_3(1690)$	1688.8 ± 2.1	1344 – 1726	7.875 – 8.125	[8]
$\rho(1700)$	1720 ± 20	1344 – 1726	7.875 – 8.125	[8]
$f_0(1710)$	1720 ± 6	1344 – 1726	7.875 – 8.125	[8]
$\pi(1800)$	1816 ± 14	1726 – 2216	8.125 – 8.375	[8; 4]
$\phi_3(1850)$	1854 ± 7	1726 – 2216	8.125 – 8.375	[8; 4]
$\pi_2(1880)$	1895 ± 16	1726 – 2216	8.125 – 8.375	[8; 4]
$f_2(1950)$	1944 ± 12	1726 – 2216	8.125 – 8.375	[8; 4]
$f_2(2100)$	2011 ± 80	1726 – 2216	8.125 – 8.375	[8; 4]
$a_4(2040)$	2001 ± 10	1726 – 2216	8.125 – 8.375	[8; 4]
$f_4(2050)$	2018 ± 11	1726 – 2216	8.125 – 8.375	[8; 4]
$f_2(2300)$	2297 ± 28	2216 – 2846	8.375 – 8.625	[8; 2]
$f_2(2340)$	2339 ± 60	2216 – 2846	8.375 – 8.625	[8; 2]
strange mesons $S = \pm 1 C = B = 0$				
$K^\pm$	493.677 ± 0.016	385 – 495	6.625 – 6.875	[7; −4]
K^0	497.614 ± 0.024	385 – 495	6.625 – 6.875	[7; −4]
$K^*(892)^\pm$	891.66 ± 0.26	815 – 1047	7.375 – 7.625	[7; 2]
$K^*(892)^0$	896.00 ± 0.25	815 – 1047	7.375 – 7.625	[7; 2]
$K_1(1270)$	1272 ± 7	1047 – 1344	7.625 – 7.875	[8; −4]
$K_1(1400)$	1403 ± 7	1344 – 1726	7.875 – 8.125	[8]
$K^*(1410)$	1414 ± 15	1344 – 1726	7.875 – 8.125	[8]
$K_0^*(1430)$	1425 ± 50	1344 – 1726	7.875 – 8.125	[8]
$K_2^*(1430)^\pm$	1425.6 ± 1.5	1344 – 1726	7.875 – 8.125	[8]
$K_2^*(1430)^0$	1432.4 ± 1.3	1344 – 1726	7.875 – 8.125	[8]
$K^*(1680)$	1717 ± 27	1344 – 1726	7.875 – 8.125	[8]
$K_2(1770)^\pm$	1773 ± 8	1726 – 2216	8.125 – 8.375	[8; 4]
$K_3^*(1780)$	1776 ± 7	1726 – 2216	8.125 – 8.375	[8; 4]
$K_2(1820)$	1816 ± 13	1726 – 2216	8.125 – 8.375	[8; 4]
$K_4^*(2045)$	2045 ± 9	1726 – 2216	8.125 – 8.375	[8; 4]
charmed mesons $S = \pm 1$				
$D^\pm$	1869.62 ± 0.20	1726 – 2216	8.125 – 8.375	[8; 4]
D^0	1864.84 ± 0.17	1726 – 2216	8.125 – 8.375	[8; 4]
$D^*(2007)^0$	2006.97 ± 0.19	1726 – 2216	8.125 – 8.375	[8; 4]
$D^*(2010)^\pm$	2010.27 ± 0.17	1726 – 2216	8.125 – 8.375	[8; 4]
$D_1(2420)^0$	2423.3 ± 1.3	2216 – 2846	8.375 – 8.625	[8; 2]
$D_2^8(2460)^0$	2461.1 ± 1.6	2216 – 2846	8.375 – 8.625	[8; 2]
$D_2^8(2460)^\pm$	2460.1 ± 3.5	2216 – 2846	8.375 – 8.625	[8; 2]

mesons	measured mass MeV/c^2	mass interval MeV/c^2	S-interval	S
charmed, strange mesons $C = S = \pm 1$				
$D_S^{\pm}$	1968.49 ± 0.34	1726 – 2216	8.125 – 8.375	[8; 4]
$D_S^{*\pm}$	2112.3 ± 0.5	1726 – 2216	8.125 – 8.375	[8; 4]
$D_{S0}^{*}(2317)^{\pm}$	2317.8 ± 0.6	2216 – 2846	8.375 – 8.625	[8; 2]
$D_{S1}(2460)^{\pm}$	2459.6 ± 0.6	2216 – 2846	8.375 – 8.625	[8; 2]
$D_{S1}(2536)^{\pm}$	2535.35 ± 0.34	2216 – 2846	8.375 – 8.625	[8; 2]
$D_{S2}(2573)^{\pm}$	2572.6 ± 0.9	2216 – 2846	8.375 – 8.625	[8; 2]
bottom mesons $B = \pm 1$				
$B^{\pm}$	5279.17 ± 0.29	4692 – 6025	9.125 – 9.375	[9; 4]
B^0	5279.50 ± 0.3	4692 – 6025	9.125 – 9.375	[9; 4]
B^*	5325.1 ± 0.5	4692 – 6025	9.125 – 9.375	[9; 4]
$B_1(5721)^0$	5723.4 ± 2.0	4692 – 6025	9.125 – 9.375	[9; 4]
$B_2^*(5747)^0$	5743 ± 5	4692 – 6025	9.125 – 9.375	[9; 4]
bottom, strange mesons $B = \pm 1, S = \pm 1$				
B_S^0	5366.3 ± 0.6	4692 – 6025	9.125 – 9.375	[9; 4]
B_S^*	5415.4 ± 1.4	4692 – 6025	9.125 – 9.375	[9; 4]
bottom, charmed mesons $B = S = \pm 1$				
$B_c^{\pm}$	6277 ± 6	6025 – 7736	9.375 – 9.625	[9; 2]
cc-mesons $B = S = \pm 1$				
$\eta_c(1S)$	2980.5 ± 1.2	2846 – 3654	8.625 – 8.875	[9; −4]
$J/psi(1S)$	3096.916 ± 0.011	2846 – 3654	8.625 – 8.875	[9; −4]
$X_{c0}(1P)$	3414.75 ± 0.31	2846 – 3654	8.625 – 8.875	[9; −4]
$X_{c1}(1P)$	3510.66 ± 0.07	2846 – 3654	8.625 – 8.875	[9; −4]
$h_c(1P)$	3525.67 ± 0.32	2846 – 3654	8.625 – 8.875	[9; −4]
$X_{c2}(1P)$	3556.20 ± 0.09	2846 – 3654	8.625 – 8.875	[9; −4]
$\eta_c(2S)$	3637 ± 4	2846 – 3654	8.625 – 8.875	[9; −4]
$\psi(2S)$	3686.09 ± 0.04	3654 – 4692	8.875 – 9.125	[9]
$\psi(3770)$	3772.92 ± 0.35	3654 – 4692	8.875 – 9.125	[9]
$X(3872)$	3872.3 ± 0.8	3654 – 4692	8.875 – 9.125	[9]
$X(3945)$	3916 ± 6	3654 – 4692	8.875 – 9.125	[9]
$\psi(4400)$	4039 ± 1	3654 – 4692	8.875 – 9.125	[9]
$\psi(4160)$	4153 ± 3	3654 – 4692	8.875 – 9.125	[9]
$\psi(4260)$	4263 ± 9	3654 – 4692	8.875 – 9.125	[9]
$\psi(4415)$	4421 ± 4	3654 – 4692	8.875 – 9.125	[9]
bb-mesons				
$Y(1S)$	9460.30 ± 0.26	7736 – 9933	9.625 – 9.875	[10; −4]
$\chi_{b0}(1P)$	9859.44 ± 0.42	7736 – 9933	9.625 – 9.875	[10; −4]
$\chi_{b1}(1P)$	9892.78 ± 0.26	7736 – 9933	9.625 – 9.875	[10; −4]
$\chi_{b2}(1P)$	9912.21 ± 0.26	7736 – 9933	9.625 – 9.875	[10; −4]
$Y(2S)$	10023.26 ± 0.31	9933 – 12754	9.875 – 10.125	[10]
$\chi_{b0}(2P)$	10232.5 ± 0.4	9933 – 12754	9.875 – 10.125	[10]
$\chi_{b1}(2P)$	10255.46 ± 0.22	9933 – 12754	9.875 – 10.125	[10]
$\chi_{b2}(2P)$	10268.65 ± 0.22	9933 – 12754	9.875 – 10.125	[10]
$Y(3S)$	10355.2 ± 0.5	9933 – 12754	9.875 – 10.125	[10]
$Y(4S)$	10579.4 ± 1.2	9933 – 12754	9.875 – 10.125	[10]
$Y(10860)$	10865 ± 8	9933 – 12754	9.875 – 10.125	[10]
$Y(11020)$	11019 ± 8	9933 – 12754	9.875 – 10.125	[10]

Table 4. The measured masses of leptons [6], the calculated S-intervals of $\frac{1}{4}$ logarithmic units width and the corresponding calculated mass-intervals.

leptons	measured mass MeV/c^2	mass interval MeV/c^2	S-interval	S
electron	$0.510998910 \pm 0.000000013$	0	0	[0]
μ	105.658367 ± 0.000004	86 – 110	5.125 – 5.375	[5; 4]
τ	1776.84 ± 0.17	1726 – 2216	8.125 – 8.375	[8; 4]

Table 5. The measured masses of gauge bosons [6], the calculated S-intervals of $\frac{1}{4}$ logarithmic units width and the corresponding calculated mass-intervals.

gauge bosons	measured mass MeV/c^2	mass interval MeV/c^2	S-interval	S
W	80398 ± 25	73395 – 94241	11.875 – 12.125	[12]
Z	$91187,6 \pm 2.1$	73395 – 94241	11.875 – 12.125	[12]