

## Gravity as Attractor Effect of Stability Nodes in Chain Systems of Harmonic Quantum Oscillators

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In this paper we apply our fractal model of matter as chain systems of harmonic quantum oscillators to the analysis of gravimetric characteristics of the Solar system and introduce a model of gravity as macroscopic cumulative attractor effect of stability nodes in chain systems of oscillating protons and electrons.

### Introduction

Gravity has still a special place in physics as it is the only interaction that is not described by a quantum theory. Nevertheless, the big  $G$  is considered to be a fundamental constant of nature, involved in the calculation of gravitational effects in Newton's law of universal gravitation and in Einstein's general theory of relativity. The currently recommended [1] value is  $G = 6.67408(31) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$  and it seems that we know  $G$  only to three significant figures.

For several objects in the Solar System, the value of the standard gravitational parameter  $\mu$  is known to greater accuracy than  $G$ . The value  $\mu$  for the Sun is the heliocentric gravitational constant and equals  $1.32712440042(1) \cdot 10^{20} \text{ m}^3 \text{ s}^{-2}$ . The geocentric gravitational constant equals  $3.986004418(8) \cdot 10^{14} \text{ m}^3 \text{ s}^{-2}$  [2]. The precision is  $10^{-8}$  because this quantity is derived from the movement of artificial satellites, which basically involves observations of the distances from the satellite to earth stations at different times, which can be obtained to high accuracy using radar or laser ranging.

However, not the  $\mu$  is directly measured, but the orbital elements of a natural or artificial satellite. For instance, the orbital elements of the Earth can be used to estimate the heliocentric gravitational constant. Already the basic solution for a circular orbit gives a good approximation:

$$\begin{aligned} \mu &= \frac{4\pi^2 R^3}{T^2} = \frac{4\pi^2 (149597870700 \text{ m})^3}{(31558149.54 \text{ s})^2} = \\ &= 1.327128 \cdot 10^{20} \text{ m}^3 \text{ s}^{-2} \end{aligned}$$

where  $R$  is the semi-major axis and  $T$  is the orbital period of the Earth. These orbital elements are directly measured, although  $\mu = GM$  is an interpretation that provides mass as source of gravity and the universality of  $G$ . Within the principle of equivalence, gravity is a universal property like inertia and does not depend on the type or scale of matter.

Though, the big  $G$  is known only from laboratory measurements of the attraction force between two known masses. The precision of those measures is only  $10^{-3}$ , because gravity appears too weak on the scale of laboratory-sized masses for to be measurable with the desired precision. However, as mentioned Quinn and Speake [3], the discrepant results may

demonstrate that we do not understand the metrology of measuring weak forces or they may signify some new physics.

On the other hand, the measured  $G$  values seem to oscillate over time [4]. It's not  $G$  itself that is varying, Anderson and coauthors proposed, but more likely something else is affecting the measurements, because the 5.9-year oscillatory period of the measured  $G$  values seems to correlate with the 5.9-year oscillatory period of Earth's rotation rate, as determined by recent Length of Day (LOD) measurements [5]. However, this hypothesis is still under discussion [6].

In 1981, Stacey, Tuck, Holding, Maher and Morris [7] reported anomalous measures of the gravity acceleration in mines. They proposed an explanation of this anomaly by introducing a short-range potential, of the Yukawa type, that overlaps the Newtonian potential and describes the intensity and the action range of a hypothetical fifth interaction. In 2005, Reginald T. Cahill [8] introduced an additional dimensionless constant that coincides with the fine structure constant and determines the strength of a new 3-space self-interaction that can explain various gravitational anomalies, such as the 'borehole anomaly' and the 'dark matter anomaly' in the rotation speeds of spiral galaxies.

Obviously, the origin of gravity and the nature of particle mass generation are key topics in modern physics and they seem to have a common future. In [9] we have introduced a fractal model of matter as a chain system of harmonic quantum oscillators and have shown that particle rest masses coincide with the eigenstates of the system. This is valid not only for hadrons, but for mesons and leptons as well. Andreas Ries [10] demonstrated that this model allows for the prediction of the most abundant isotope of a given chemical element. Already in [11] we could show that scale invariance is a fundamental property of this model. On this background we proposed quantum scaling as model of mass generation [12].

Our model of matter also provides a good approximation of the mass distribution of large celestial bodies in the Solar system [13]. Metric characteristics of celestial bodies can be understood as macroscopic quantized eigenstates in chain systems of oscillating protons and electrons [14].

In [15] we have calculated the model masses of new planets in the Solar system and in [16, 17] were estimated the orbital elements of these hypothetical bodies. Our calculations



Fig. 1: The canonical projection of  $\mathcal{F}$  (natural logarithmic representation).

correspond well with the hypothesis of Batygin and Brown [18] about a new gas giant called “planet 9” and with the hypothesis of Volk and Malhotra [19] about a Mars-to-Earth mass “planet 10” beyond Pluto.

Our model allows us to see a connection between the stability of the Solar system and the stability of the electron and proton and consider global scaling as a forming factor of the Solar system. This may be of cosmological significance.

In this paper we apply our model of matter to the analysis of gravimetric characteristics of large bodies of the Solar system and propose an interpretation of gravity as macroscopic cumulative attractor effect of stability nodes in chain systems of oscillating protons and electrons.

## Methods

In [11] we have shown that the set of natural frequencies of a chain system of similar harmonic oscillators coincides with a set of finite continued fractions  $\mathcal{F}$ , which are natural logarithms:

$$\ln(\omega_{jk}/\omega_{00}) = n_{j0} + \frac{z}{n_{j1} + \frac{z}{n_{j2} + \dots + \frac{z}{n_{jk}}}} = [z, n_{j0}; n_{j1}, n_{j2}, \dots, n_{jk}] = \mathcal{F}, \quad (1)$$

where  $\omega_{jk}$  is the set of angular frequencies and  $\omega_{00}$  is the fundamental frequency of the set. The denominators are integer:  $n_{j0}, n_{j1}, n_{j2}, \dots, n_{jk} \in \mathbb{Z}$ , the cardinality  $j \in \mathbb{N}$  of the set and the number  $k \in \mathbb{N}$  of layers are finite. In the canonical form, the numerator  $z$  equals 1.

For finite continued fractions  $\mathcal{F}$  (1), ranges of high distribution density (nodes) arise near reciprocal integers  $1, 1/2, 1/3, 1/4, \dots$  which are the attractor points of the distribution.

Any finite continued fraction represents a rational number [20]. Therefore, all natural frequencies  $\omega_{jk}$  in (1) are irrational, because for rational exponents the natural exponential function is transcendental [21]. It is probable that this circumstance provides for high stability of an oscillating chain system because it prevents resonance interaction between the elements of the system [22]. Already in 1987 we have applied continued fractions of the type  $\mathcal{F}$  (1) as criterion of stability in engineering [23, 24].

In the case of harmonic quantum oscillators, the continued fractions  $\mathcal{F}$  (1) not only define fractal sets of natural angular frequencies  $\omega_{jk}$ , angular accelerations  $a_{jk} = c \cdot \omega_{jk}$ , oscillation periods  $\tau_{jk} = 1/\omega_{jk}$  and wavelengths  $\lambda_{jk} = c/\omega_{jk}$  of

the chain system, but also fractal sets of energies  $E_{jk} = \hbar \cdot \omega_{jk}$  and masses  $m_{jk} = E_{jk}/c^2$  which correspond with the eigenstates of the system. For this reason, we call the continued fraction  $\mathcal{F}$  (1) the “fundamental fractal” of eigenstates in chain systems of harmonic quantum oscillators.

In the canonical form ( $z = 1$ ) of the fundamental fractal  $\mathcal{F}$  (1), shorter continued fractions correspond with more stable eigenstates of a chain system of harmonic oscillators. Therefore, integer logarithms represent the most stable eigenstates (main attractor nodes).

Normal matter is formed by nucleons and electrons because they are exceptionally stable. Furthermore, protons and neutrons have similar rest masses (the difference being only 0.14 percent). This allows us to interpret the proton and the neutron as similar quantum oscillators with regard to their rest masses. Therefore, in [12, 14] we have introduced a fractal model of matter as a chain system of oscillating protons and electrons.

Table 1 shows the basic set of electron and proton units that can be considered as a fundamental metrology ( $c$  is the speed of light in vacuum,  $\hbar$  is the reduced Planck constant).

Table 1: The basic set of physical properties of the electron and proton. Data taken from Particle Data Group [25]. Frequencies, oscillation periods, accelerations and the proton wavelength are calculated.

property	electron	proton
rest mass $m$	$9.10938356(11) \cdot 10^{-31}$ kg	$1.672621898(21) \cdot 10^{-27}$ kg
energy $E = mc^2$	$0.5109989461(31)$ MeV	$938.2720813(58)$ MeV
angular frequency $\omega = E/\hbar$	$7.76344071 \cdot 10^{20}$ Hz	$1.42548624 \cdot 10^{24}$ Hz
angular oscillation period $\tau = 1/\omega$	$1.28808867 \cdot 10^{-21}$ s	$7.01515 \cdot 10^{-25}$ s
wavelength $\lambda = c/\omega$	$3.8615926764(18) \cdot 10^{-13}$ m	$2.1030891 \cdot 10^{-16}$ m
angular acceleration $a = c\omega$	$2.327421 \cdot 10^{29}$ ms <sup>-2</sup>	$4.2735 \cdot 10^{32}$ ms <sup>-2</sup>

The natural logarithm of the proton-to-electron mass ratio is approximately 7.5 and consequently, the fundamental fractal  $\mathcal{F}$  calibrated on the proton will be shifted by 7.5 logarithmic units relative to the  $\mathcal{F}$  calibrated on the electron:

$$\ln \frac{1.672621898 \cdot 10^{-27} \text{ kg}}{9.10938356 \cdot 10^{-31} \text{ kg}} \approx 7.5$$

We hypothesize that scale invariance based on the fundamental fractal  $\mathcal{F}$  (1), calibrated on the metric properties of the proton and electron, is a universal characteristic of organized matter. This hypothesis we have called ‘global scaling’ [14, 26].

## Results

In [12] we have shown that the Planck mass coincides with the main attractor node [44;  $\infty$ ] of the  $\mathcal{F}$  (1) calibrated on the proton:

$$\ln \frac{m_{\text{Planck}}}{m_{\text{proton}}} = \ln \frac{2.17647 \cdot 10^{-8}}{1.6726219 \cdot 10^{-27}} = 44.01$$

This circumstance allows us to calculate the big  $G$  from the proton rest mass:

$$G = \frac{\hbar c}{m_p^2} \exp(-88) = 6.8420676 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

The calculated  $G$  value is larger than the currently recommended by CODATA [1], although the published [27,28] values of  $G$  show immense variations and some recent measurements of high precision deliver, in fact, larger values than the recommended.

Applying our model (1), we can see that the Solar equatorial surface gravity acceleration  $g_{\text{Sun}} = 274 \text{ m/s}^2$  corresponds with a main attractor node of the  $\mathcal{F}$  (1) calibrated on the angular acceleration of the electron  $a_{\text{electron}} = 2.327421 \cdot 10^{29} \text{ ms}^{-2}$  (see table 1). In fact, the logarithm of the electron-to-Solar gravity acceleration ratio is close to an integer:

$$\ln \frac{a_{\text{electron}}}{g_{\text{Sun}}} = \ln \frac{2.327421 \cdot 10^{29} \text{ ms}^{-2}}{274 \text{ ms}^{-2}} = 62.00$$

This coincidence supports our hypothesis of global scaling and allows us to understand that the current amount of the surface gravity acceleration of the Sun is not casual, but an essential aspect of stability of the chain system of quantum oscillators that appears as the star we call 'Sun'.

Also the current amount of the Solar mass we recognise as criterion of stability, because it corresponds to a main attractor node of the  $\mathcal{F}$  (1) calibrated on the electron. In fact, the natural logarithm of the Sun-to-electron mass ratio is close to an integer number:

$$\ln \frac{M_{\text{Sun}}}{m_{\text{electron}}} = \ln \frac{1.9884 \cdot 10^{30} \text{ kg}}{9.10938356 \cdot 10^{-31} \text{ kg}} = 138.94$$

Furthermore, the main attractor node [62;  $\infty$ ] of the  $\mathcal{F}$  (1) calibrated on the electron corresponds with the node [69; 2] calibrated on the proton that is half of the logarithm of the Solar-to-electron mass ratio:  $69.5 = 139/2$ . This allows us to write down an equation that connects the Sun-to-electron mass ratio with the proton-to-Solar surface gravity acceleration ratio:

$$\frac{M_{\text{Sun}}}{m_{\text{electron}}} = \left( \frac{a_{\text{proton}}}{g_{\text{Sun}}} \right)^2$$

As well, the correspondence of the current radius of the Sun with a main attractor node (integer logarithm) of the  $\mathcal{F}$  (1)

calibrated on the electron now we can understand as additional criterion of stability of the Sun:

$$\ln \frac{R_{\text{Sun}}}{\lambda_{\text{electron}}} = \ln \frac{6.96407 \cdot 10^8 \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}} = 48.95$$

The logarithm of the proton-to-Jupiter surface gravity acceleration ratio is also close to an integer:

$$\ln \frac{a_{\text{proton}}}{g_{\text{Jupiter}}} = \ln \frac{4.2735 \cdot 10^{32} \text{ ms}^{-2}}{24.79 \text{ ms}^{-2}} = 71.92$$

Jupiter's body mass coincides with the main attractor node [132;  $\infty$ ] of the electron-calibrated  $\mathcal{F}$  (1):

$$\ln \frac{M_{\text{Jupiter}}}{m_{\text{electron}}} = \ln \frac{1.8986 \cdot 10^{27} \text{ kg}}{9.10938356 \cdot 10^{-31} \text{ kg}} = 131.98$$

The surface gravity accelerations of Saturn ( $10.4 \text{ m/s}^2$ ), Uranus ( $8.7 \text{ m/s}^2$ ), Neptune ( $11.1 \text{ m/s}^2$ ), Earth ( $9.81 \text{ m/s}^2$ ) and Venus ( $8.87 \text{ m/s}^2$ ) approximate the main attractor node [73;  $\infty$ ] of the  $\mathcal{F}$  (1) calibrated on the proton:

$$\ln \frac{a_{\text{proton}}}{g_{\text{Venus}}} = \ln \frac{4.2735 \cdot 10^{32} \text{ ms}^{-2}}{8.87 \text{ ms}^{-2}} = 72.95$$

The mass of Venus corresponds to the main attractor node [126;  $\infty$ ] of the electron-calibrated  $\mathcal{F}$  (1):

$$\ln \frac{M_{\text{Venus}}}{m_{\text{electron}}} = \ln \frac{4.8675 \cdot 10^{24} \text{ kg}}{9.10938356 \cdot 10^{-31} \text{ kg}} = 126.01$$

Finally, the surface gravity accelerations of Mercury and Mars ( $3.71 \text{ m/s}^2$ ) approximate the main attractor node [74;  $\infty$ ] of the  $\mathcal{F}$  (1) calibrated on the proton:

$$\ln \frac{a_{\text{proton}}}{g_{\text{Mars}}} = \ln \frac{4.2735 \cdot 10^{32} \text{ ms}^{-2}}{3.71 \text{ ms}^{-2}} = 73.83$$

The body mass of Mars corresponds to the main attractor node [124;  $\infty$ ] of the  $\mathcal{F}$  (1) calibrated on the electron:

$$\ln \frac{M_{\text{Mars}}}{m_{\text{electron}}} = \ln \frac{6.4171 \cdot 10^{23} \text{ kg}}{9.10938356 \cdot 10^{-31} \text{ kg}} = 123.99$$

In [14] we have shown that the body masses, the rotation and orbital periods of the planets and the Sun are quantized. They follow the sequence of attractor nodes of stability of the fundamental fractal  $\mathcal{F}$  (1). Now we can affirm that the surface gravity accelerations of the planets and the Sun are quantized as well. The surface gravity accelerations of the planets correspond with the main attractor nodes [72;  $\infty$ ], [73;  $\infty$ ], [74;  $\infty$ ] of the  $\mathcal{F}$  (1) calibrated on the proton while the surface gravity acceleration of the Sun corresponds with the main attractor node [62;  $\infty$ ] of the  $\mathcal{F}$  (1) calibrated on the electron.

Considering that the angular acceleration of the electron is  $a_{\text{electron}} = c\omega_{\text{electron}}$ , we can express the Solar surface gravity acceleration in terms of the speed of light

$$g_{\text{Sun}} = c\omega_{\text{Sun}}$$

and receive the angular oscillation period  $1/\omega_{\text{Sun}} = 12.7$  sidereal days that is the first harmonic of the equatorial rotation period 25.4 days of the Sun. This coincidence suggests to analyse also the gravity accelerations of the planets in terms of the speed of light.

If we express the Earth surface gravity acceleration  $g = 9.8 \text{ ms}^{-2}$  in terms of the speed of light, we receive an oscillation period of  $c/g = 355$  sidereal days that is in the range of the Earth orbital period and coincides perfectly with the attractor node [63; 2] of the  $\mathcal{F}$  (1) calibrated on the electron oscillation period  $2\pi\tau_{\text{electron}} = 8.0933 \cdot 10^{-21} \text{ s}$ :

$$2\pi\tau_{\text{electron}} \exp(63.5) = 355 \text{ days}$$

The period of 355 days coincides with 12 synodic lunar months, the lunar year. The surface gravity accelerations of Saturn ( $10.4 \text{ m/s}^2$ ), Uranus ( $8.7 \text{ m/s}^2$ ), Neptune ( $11.1 \text{ m/s}^2$ ) and Venus ( $8.87 \text{ m/s}^2$ ) are of the same range and consequently, they approximate the same attractor node [63; 2].

The surface gravity acceleration of Saturn  $g_{\text{Saturn}} = 10.4 \text{ m/s}^2$  corresponds with an oscillation period of  $c/g_{\text{Saturn}} = 334$  sidereal days that is in the range of the duration of lightning storms on Saturn which appear once every 30 Earth years. The lightning storm of 2009 on Saturn in the planet's southern hemisphere lasted over 334 days [29].

Mars and Mercury have similar surface gravity accelerations of about  $3.7 \text{ ms}^{-2}$  that corresponds to an oscillation period of  $c/3.7 \text{ ms}^{-2} = 938$  sidereal days near the attractor node [64; 2] of the  $\mathcal{F}$  (1) calibrated on the electron:

$$2\pi\tau_{\text{electron}} \exp(64.5) = 966 \text{ days}$$

The sidereal rotation period of Mars is 24.62278 hours and coincides perfectly to the main node [67;  $\infty$ ] of the proton-calibrated  $\mathcal{F}$  (1):

$$\ln \frac{\tau_{\text{Mars}}}{\tau_{\text{proton}}} = \ln \frac{24.62278 \cdot 3600 \text{ s}}{7.01515 \cdot 10^{-25} \text{ s}} = 67.00$$

In addition, the orbital period of Mars 686.971 days meets precisely the condition of global scaling:

$$\ln \frac{T_{\text{Mars}}}{\tau_{\text{electron}}} = \ln \frac{686.971 \cdot 86164 \text{ s}}{1.28808867 \cdot 10^{-21} \text{ s}} = 66.00$$

The surface gravity acceleration of Jupiter  $g_{\text{Jupiter}} = 24.79 \text{ ms}^{-2}$  corresponds to an oscillation period of  $c/g_{\text{Jupiter}} = 140$  sidereal days near the main attractor node of the  $\mathcal{F}$  calibrated on the electron:

$$2\pi\tau_{\text{electron}} \exp(62.5) = 131 \text{ days}$$

The sidereal rotation period of Jupiter is 9.925 hours and corresponds with the main attractor node [66;  $\infty$ ] of the proton  $\mathcal{F}$  (1):

$$\ln \frac{\tau_{\text{Jupiter}}}{\tau_{\text{proton}}} = \ln \frac{9.9251 \cdot 3600 \text{ s}}{7.01515 \cdot 10^{-25} \text{ s}} = 66.10$$

Jupiter's orbital period of 4332.59 days fulfils the conditions of global scaling very precisely:

$$\ln \frac{T_{\text{Jupiter}}}{2\pi\tau_{\text{electron}}} = \ln \frac{4332.59 \cdot 86164 \text{ s}}{2\pi \cdot 1.28808867 \cdot 10^{-21} \text{ s}} = 66.00$$

When the logarithm of the sidereal rotation period of Jupiter slows down to [66;  $\infty$ ], the orbital-to-rotation period ratio of Jupiter can be described by the equation:

$$\frac{T_{\text{Jupiter}}}{\tau_{\text{Jupiter}}} = \frac{2\pi\tau_{\text{electron}}}{\tau_{\text{proton}}}$$

We can see that both the orbital periods of Jupiter and Mars correspond with the main attractor node [66;  $\infty$ ] of stability, but in the case of Jupiter with the electron oscillation period as fundamental and in the case of Mars with the electron angular oscillation period as fundamental. Therefore, both orbital periods are simply connected by  $2\pi$ :

$$T_{\text{Jupiter}} = 2\pi T_{\text{Mars}}$$

Also these circumstances support our model of matter as chain system of harmonic quantum oscillators and our hypothesis of global scaling.

## Conclusion

Applying our fractal model of matter as chain system of harmonic quantum oscillators to the analysis of gravimetric characteristics of large bodies of the Solar system we did show that the surface gravity accelerations of the planets and the Sun are quantized and correspond to nodes of stability in chain systems of oscillating protons and electrons and therefore, they can be estimated without any information about the masses or sizes of the celestial bodies.

Furthermore, the quantized surface gravity accelerations of the planets and the Sun seem to be connected with their quantized orbital and rotation periods.

We presume that the accretion of gravitational mass is a macroscopic cumulative attractor effect of stability nodes in chain systems of oscillating protons and electrons. From this point of view, Newton's constant of gravitation defines the corresponding amount of gravitational mass a given attractor node can accumulate.

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