On the Cosmological Significance of Euler's Number

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The paper derives and exemplifies the stabilizing significance of Euler's number in particle physics, biophysics, geophysics, astrophysics and cosmology.

Introduction

Natural systems are highly complex and at the same time they impress us with their lasting stability. For instance, the solar system hosts at least 800 thousand orbiting each other bodies. If numerous bodies are gravitationally bound to one another, classic models predict long-term highly unstable states [1, 2]. Indeed, considering the destructive potential of resonance, how this huge system can be stable?

In the following we will see that the difference between rational, irrational algebraic and transcendental numbers is not only a mathematical task. It is also an essential aspect of stability in complex systems.

Actually, if the ratio of any two orbital periods would be a rational number, periodic gravity interaction would progressively rock the orbital movements and ultimately cause a resonance disaster that could destabilize the solar system. Therefore, lasting stability in complex dynamic systems is possible only if whole number frequency ratios can be avoided.

Obviously, irrational numbers cannot be represented as a ratio of whole numbers and consequently, they should not cause destabilizing resonance interaction [3, 4].

Though, algebraic irrational numbers like $\sqrt{2}$ do not compellingly prevent resonance, because they can be transformed into rational numbers by multiplication. In the case of $\sqrt{2}$ as a frequency ratio, every even harmonic is integer, because $\sqrt{2} \cdot \sqrt{2} = 2$.

However, there is a type of irrational numbers called transcendental which are not roots of whole or rational numbers. They cannot be transformed into rational or whole numbers by multiplication and consequently, they do not provide resonance interaction.

Actually, frequencies of real periodical processes are not constant. Their temporal change is described by accelerations, the derivatives of the frequencies. Naturally, accelerations are not constant either.

Surprisingly, there is only one transcendental number that inhibits resonance also regarding accelerations and any other derivatives: it is Euler's number $e = 2.71828\ldots$, because it is the basis of the natural exponential function e^x, the only function that is the derivative of itself.

In this way, the number continuum provides the solution for lasting stability in systems of any degree of complexity. The solution is given a priori: frequency ratios equal to Euler's number, its integer powers or roots are always transcendental [5] and inhibit destructive resonance interaction regarding all derivatives of the interconnected periodic processes. Therefore, we expect that periodic processes in stable systems show frequency ratios close to integer powers of Euler's number or its roots. Consequently, the logarithms of the frequency ratios should be close to integer 0, 1, 2, 3, 4, ... or rational values $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

In the following we will exemplify our hypothesis in particle physics, biophysics, geophysics, astrophysics and cosmology. We start with the solar system.

Euler's number stabilizes the solar system

Let us analyze the ratios of the orbital periods of some planets. Saturn's sidereal orbital period [6] equals 10759.22 days, that of Uranus is 30688.5 days. The natural logarithm of the ratio of their orbital periods is close to 1:

$$\ln\left(\frac{30688.5}{10759.22}\right) = 1.05$$

Jupiter's sidereal orbital period equals 4332.59 days, that of the planetoid Ceres is 1681.63 days. The natural logarithm of the ratio of their orbital periods is also close to 1:

$$\ln\left(\frac{4332.59}{1681.63}\right) = 0.95$$

Not only neighboring orbits show Euler ratios, but far apart from each other orbits do this as well. Pluto's sidereal orbital period is 90560 days, that of Venus is 224.701 days. The natural logarithm of the ratio of their orbital periods equals 6:

$$\ln\left(\frac{90560}{224.701}\right) = 6.00$$

In [7] we have analyzed the orbital periods of the largest bodies in the solar system including the moon systems of Jupiter, Saturn, Uranus and Neptune, as well as the exoplanetary systems Trappist 1 and Kepler 20. In the result we can assume that the stability of all these orbital systems is given by the transcendence of Euler's number and its roots.

Euler's number stabilizes biological rhythms

Biological processes are of highest complexity and their lasting stability is of vital importance. Therefore, we expect that established periodical biological processes show Euler frequency ratios. In fact, at resting state, the majority of adults prefer to breath [8] with an average frequency of 15 inhale-exhale sequences per minute, while their heart rate [9] is close to 67 beats per minute. The natural logarithm of the ratio of these frequencies equals $1+\frac{1}{2}$:

$$\ln\left(\frac{67}{15}\right) = 1.50$$

Mammals including human show electrical brain activity [10] of the Theta type in the frequency range between 3 and 7 Hz, of Alpha type between 8 and 13 Hz and Beta type between 14 and 34 Hz. Below 3 Hz the brain activity is of the Delta type, and above 34 Hz the brain activity changes to Gamma.

The frequencies 3 Hz, 8 Hz, 13 Hz and 34 Hz define the boundaries. The logarithms of their ratios are close to integer and half values:

$$\ln\left(\frac{8}{3}\right) = 0.98 \qquad \ln\left(\frac{13}{8}\right) = 0.49 \qquad \ln\left(\frac{34}{13}\right) = 0.96$$

In [11] we have analyzed various biological frequency ranges and assume that their stability is given by the transcendence of Euler's number and its roots.

Euler's number stabilizes the atom

The most stable systems we know are of atomic scale. Proton and electron form stable atoms, the structural elements of matter. The lifespans of the proton and electron surpass everything that is measurable, exceeding 10^{30} years. No scientist ever witnessed the decay of a proton or an electron. What is the secret of their eternal stability?

In standard particle physics, the electron is stable because it is the least massive particle with non-zero electric charge. Its decay would violate charge conservation. Indeed, this answer only readdresses the question. Why then is the elementary electric charge so stable?

In theoretical physics, the proton is stable, because it is the lightest baryon and the baryon number is conserved. Indeed, also this answer only readdresses the question. Why then is the proton the lightest baryon? To answer this question, the standard model introduces quarks which violate the integer quantization of the elementary electric charge.

Now let us proof our hypothesis of Euler's number as universal stabilizer and analyze the proton-to-electron ratio 1836.152674 that is considered as fundamental physical constant [12]. It has the same value for the natural frequencies, oscillation periods, wavelengths, rest energies and rest masses of the proton and electron. In fact, the natural logarithm is close to seven and a half:

$$\ln(1836.152674) = 7.51$$

This result suggests the assumption that the stability of the proton and electron comes from the number continuum, more specifically, from the transcendence of Euler's number, its integer powers and roots. In [13] we have analyzed the mass distribution of hadrons, mesons, leptons, the W/Z and Higgs bosons and proposed fractal scaling by Euler's number and its roots as model of particle mass generation [14]. In this model, the W-boson mass 80385 MeV/c^2 and the Z-boson mass 91188 MeV/c^2 appear as the 12 times scaled up electron rest mass 0.511 MeV/c^2:

$$\ln\left(\frac{80385}{0.511}\right) = 11.97 \qquad \ln\left(\frac{91188}{0.511}\right) = 12.09$$

In [15] Andreas Ries did apply fractal scaling by Euler's number to the analysis of particle masses and in [16] he demonstrated that this method allows for the prediction of the most abundant isotopes.

Global scaling based on Euler's number

Our hypothesis about Euler's number as universal stabilizer allows us to calculate Pluto's orbital period from that of Venus multiplying 6 times by Euler's number:

$$Venus\ orbital\ period \cdot e^6 = Pluto\ orbital\ period$$

Each time we multiply by Euler's number, we get an orbital period of a planet in the following sequence: Mars, Ceres, Jupiter, Saturn, Uranus and Pluto. Dividing by Euler's number, we get close to the orbital period of Mercury. Earth's orbital period we get multiplying by the square root of Euler's number. The same is valid for Neptune relative to Uranus.

Euler's number and its roots are universal scaling factors that inhibit resonance and in this way, stabilize periodical processes bound in a chain system. Pluto's orbital period can be seen as the 6 times scaled up by Euler's number orbital period of Venus or as the 3 times scaled up by Euler's number orbital period of Jupiter.

In the same way, the oscillation period of the electron can be seen as the $7 + \frac{1}{2}$ times scaled up oscillation period of the proton. Here it is important to understand that only scaling by Euler's number and its roots inhibits resonance interaction and provides lasting stability of the interconnected processes.

Now we could ask the question: Starting with the electron oscillation period, if we continue to scale up always multiplying by Euler's number, will we meet the orbital period, for instance, of Jupiter?

Actually, it is true. If we multiply the electron natural oscillation period 66 times by Euler's number, we meet exactly the orbital period of Jupiter:

$$electron\ oscillation\ period \cdot e^{66} = Jupiter\ orbital\ period$$

The oscillation period of the electron has a duration of $2\pi \cdot 1.288089 \cdot 10^{-21} s = 8.0933 \cdot 10^{-21}$ s. Jupiter' orbital period takes 4332.59 days = $3.7331 \cdot 10^8$ s. In fact, the natural logarithm of the ratio of Jupiter' orbital period to the electron oscillation period equals 66:

$$\ln\left(\frac{3.7331 \cdot 10^8\ \mathrm{s}}{8.0933 \cdot 10^{-21}\ \mathrm{s}}\right) = 66.00$$

PROPERTY	ELECTRON	PROTON
rest energy E	0.5109989461(31) MeV	938.2720813(58) MeV
rest mass $m = E/c^2$	$9.10938356(11) \cdot 10^{-31}$ kg	$1.672621898(21) \cdot 10^{-27}$ kg
blackbody temperature $\mathrm{T} = E/k$	$5.9298446 \cdot 10^{9}$ K	$1.08881 \cdot 10^{13}$ K
angular frequency $\omega = E/\hbar$	$7.763441 \cdot 10^{20}$ Hz	$1.425486 \cdot 10^{24}$ Hz
angular oscillation period $\tau = 1/\omega$	$1.288089 \cdot 10^{-21}$ s	$7.01515 \cdot 10^{-25}$ s
angular wavelength $\lambda = c/\omega$	$3.8615926764(18) \cdot 10^{-13}$ m	$2.103089 \cdot 10^{-16}$ m

Table 1: The basic set of physical properties of the electron and proton (c is the speed of light in a vacuum, $\hbar$ is the reduced Planck constant, k is the Boltzmann constant). Data taken from Particle Data Group [12]. Frequencies, oscillation periods, temperatures and the proton wavelength are calculated.

Forming atoms and molecules, proton and electron are substantial components of biological organisms as well. Through scaling, Euler's number stabilizes biological processes down to the subatomic scales of the electron and proton. Dividing the angular frequency of the electron 48 times by Euler's number, we get the average adult human heart rate:

$$electron\ angular\ frequency\ /\ e^{48} = adult\ human\ heart\ rate$$

In fact, the natural logarithm of the ratio of the average adult human heart rate 67/min to the electron angular frequency (tab. 1) equals -48:

$$\ln\left(\frac{67/60}{7.763441 \cdot 10^{20}}\right) = -48.00$$

In a similar way, dividing the angular frequency of the proton 57 times by Euler's number, we get the average adult human respiratory rate:

$$proton\ angular\ frequency\ /\ e^{57} = adult\ respiratory\ rate$$

In fact, the natural logarithm of the ratio of the average adult human resting respiratory rate 15/min to the proton angular frequency (tab. 1) equals -57:

$$\ln\left(\frac{15/60}{1.425486 \cdot 10^{24}}\right) = -57.00$$

Through scaling by Euler's number, systemically important processes of very different scales avoid resonance. In [17] we have shown how the metric characteristics of biological systems are embedded in the solar system and prevented from destructive proton and electron resonance through scaling by Euler's number.

The exceptional stability of the electron and proton predestinates them as the forming elements of baryonic matter and makes them omnipresent in the universe. Therefore, the prevention of complex systems from electron or proton resonance is an essential condition of their lasting stability.

This uniqueness of the electron and proton predispose their physical characteristics (tab. 1) to be treated as natural metrology, completely compatible with Planck units. Originally proposed in 1899 by Max Planck, they are also known as natural units, because they origin only from properties of nature and not from any human construct. Natural units are based only on the properties of space-time.

Max Planck wrote [18] that these units, "regardless of any particular bodies or substances, retain their importance for all times and for all cultures, including alien and non-human, and can therefore be called natural units of measurement".

If now we express Jupiter's body mass in electron masses, we can see how Euler's number prevents Jupiter from destructive electron resonance. In fact, the logarithm of the Jupiter-to-electron mass ratio is close to the integer 132:

$$\ln\left(\frac{1.8986 \cdot 10^{27}\ \mathrm{kg}}{9.10938 \cdot 10^{-31}\ \mathrm{kg}}\right) = 131.98$$

As we have seen already, the natural logarithm of the ratio of Jupiter's orbital period to the electron oscillation period equals 66 that is 132/2.

The same is valid for Venus. The natural logarithm of the ratio of Venus' orbital period 224.701 days = $1.9361 \cdot 10^7$ s to the electron oscillation period is close to the integer 63:

$$\ln\left(\frac{1.9361 \cdot 10^{7}\ \mathrm{s}}{8.0933 \cdot 10^{-21}\ \mathrm{s}}\right) = 63.04$$

At the same time, the logarithm of the Venus-to-electron mass ratio is close to the integer 126 that is $2 \cdot 63$:

$$\ln\left(\frac{4.8675 \cdot 10^{24}\ \mathrm{kg}}{9.10938 \cdot 10^{-31}\ \mathrm{kg}}\right) = 126.01$$

For Jupiter and Venus, now we can write down an equation that connects the body mass M with the orbital period T:

$$\left(\frac{T}{\tau_{electron}}\right)^2 = \frac{M}{m_{electron}}$$

In [19, 20] we have shown that mass-orbital scaling arises as a consequence of macroscopic quantization in chain systems of harmonic quantum oscillators and can be understood as fractal equivalent of the Hooke's law. Saturn's moon system demonstrates square root mass-orbital scaling for one and the same body, like in the case of Jupiter and Venus. The moon systems of Jupiter and Uranus show, that mass-orbital scaling can be valid also for couples of different bodies. This may mean that the orbital period of a given body is not always a function of its own mass, but depends on the mass distribution in the whole system.

In [21] we have shown how global scaling by Euler's number determines the masses, sizes, orbital and rotation periods, orbital velocities and surface gravity accelerations of the largest bodies in the solar system.

Not only the bodies of Jupiter and Venus are prevented from destructive electron resonance, but the Sun as well. In fact, the logarithm of the Sun-to-electron mass ratio is close to the integer 139:

$$\ln\left(\frac{1.9884 \cdot 10^{30}\ \mathrm{kg}}{9.10938 \cdot 10^{-31}\ \mathrm{kg}}\right) = 138.94$$

In this way, the body mass of Jupiter is the 7 times scaled down by Euler's number body mass of the Sun. The body masses of Neptune and Uranus appear as the 3 times scaled down by Euler's number body mass of Jupiter.

Scaling down by Euler's number another 3 times, we get the body mass of Venus. Again scaling down by Euler's number 2 times, we get the body mass of Mars. Scaling down by Euler's number 4 times, we get the body mass of Pluto, then dividing always by Euler's number we get the body masses of Haumea and Charon.

In [22] we did show that global scaling by Euler's number can be seen as stabilizing mechanism of planetary atmospheres that determines their stratification. In [23,24] we have applied scaling by Euler's number in engineering and developed methods of resonance inhibition and stabilization in ballistics, aerodynamics and mechanics.

Euler's number stabilizes the universe

Having analysed the solar system, now we venture into more distant regions of the Milky Way. However, we have to consider that distance measurement by parallax triangulation is precise enough only up to 500 light years. With the increase of the distances, indirect methods are applied blurring the difference between facts and model claims.

Currently there is no precise measurement of the distance to the Galactic Center, but 26,000 light years = $2.46 \cdot 10^{20}$ m seems an accepted estimation [25]. The natural logarithm of this distance divided by the proton wavelength (tab. 1) is close to the integer 83:

$$\ln\left(\frac{R_{\mathrm{GC-Sun}}}{\lambda_{\mathrm{proton}}}\right) = \ln\left(\frac{2.46 \cdot 10^{20}\ \mathrm{m}}{2.103089 \cdot 10^{-16}\ \mathrm{m}}\right) = 83.05$$

If the current measurement is correct, it would mean that the solar system orbits the Galactic Center at a distance that avoids resonance interaction with it. Good for us.

The Andromeda galaxy M31 seems to be at a distance of 2.5 million ly = $2.365 \cdot 10^{22}$ m [26] away from the Milky Way (MW). The natural logarithm of this distance divided by the electron wavelength (tab. 1) is close to the integer 80:

$$\ln\left(\frac{R_{\mathrm{MW-M31}}}{\lambda_{\mathrm{electron}}}\right) = \ln\left(\frac{2.365 \cdot 10^{22}\ \mathrm{m}}{3.861593 \cdot 10^{-13}\ \mathrm{m}}\right) = 80.10$$

For reaching the island of stability that corresponds with the integer logarithm 80, the M31-to-MW distance has to decrease by 240,000 ly down to 2.26 million light years:

$$\lambda_{\mathrm{electron}} \cdot e^{80} = 2.26 \cdot 10^{6}\ \mathrm{ly}$$

They seem to do exactly this. M31 is approaching (more precisely, 2.5 million years ago was approaching) the Milky Way at about 100 kilometers per second, as indicated by blueshift measurements [27]. If this velocity is constant, the current distance to M31 should be already 1,000 light years shorter than the 2.5 million years old distance we can measure today.

Standard model calculations expect that both galaxies will collide in a few billion years [27]. Considering the stabilizing function of Euler's number, we expect that after reaching the integer logarithm 80, the approach will be finished and the distance between both galaxies will be stabilized at 2.26 million light years. In this way, the consideration of Euler's number as resonance inhibitor and universal stabilizer can modify predictions completely.

The cosmic microwave background radiation (CMBR) is traditionally interpreted as a remnant from an early stage of the observable universe when stars and planets didn't exist yet, and the universe was denser and much hotter. Admittedly, there are alternative models [28] in development proposing explanations for the CMBR which do not implicate standard cosmological scenarios. However, traditionally CMBR data is considered as critical to cosmology since any proposed model of the universe must explain this radiation.

If this cosmic background process is stable, its average temperature 2.725 Kelvin [29] should correspond with an integer power of Euler's number. In fact, the CMBR-to-proton blackbody temperature ratio is close to the logarithm -29:

$$\ln\left(\frac{\mathrm{T}_{\mathrm{CMBR}}}{\mathrm{T}_{\mathrm{proton}}}\right) = \ln\left(\frac{2.725\ \mathrm{K}}{1.08881 \cdot 10^{13}\ \mathrm{K}}\right) = -29.01$$

In this way, the cosmic background seems to be stable, and the current temperature of the CMBR is not accidental.

We assume that global scaling by Euler's number stabilizes the whole universe [30], from the atoms up to the galaxies and the intergalactic space. In this case, any linear (non-logarithmic) observation of very large-scale structures will discover a scaling-up-effect that appears as exponential expansion of the universe. At the same time, any linear observation of very small-scale structures will discover a scaling-down-effect that appears as exponential compression down to an apparent spacetime singularity.

Conclusion

The consideration of Euler's number as resonance inhibitor and universal stabilizer adds a new aspect to our comprehension of the evolution of the universe, explaining not only the stability of the solar orbital system, but also the stability of its trajectory through the galaxy.

On the example of the M31-MW approach we demonstrated how the consideration of Euler's number as stabilizer can modify predictions completely. Applying global scaling by Euler's number to planetary systems, we can identify stabilized astrophysical processes and predict the evolution of systems that are still in formation.

We have shown that the current cosmic background temperature is not accidental and manifests the cosmological significance of Euler's number as well.

Stabilizing the proton-to-electron ratio, Euler's number provides the formation of atoms. Euler's number stabilizes biological frequency ranges down to the subatomic scale and embeds them in the dynamics of the solar system.

Finally, the apparent expansion of the universe could turn out to be a compelling consequence of the stabilizing role of Euler's number and its integer powers.

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