Physics of Transcendental Numbers on the Origin of Astrogeophysical Cycles

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Transcendental ratios of physical quantities can provide stability in complex dynamic systems because they inhibit the occurrence of destabilizing resonance between the elements of the system. In this paper we analyze recently discovered astrophysical and geophysical cycles in order to verify this numeric-physical paradigm.

Introduction

The Solar system behaves like a precise chronometer. Indeed, the orbital and rotational periods of the planets, planetoids and large moons are exceptionally stable. In view of the huge number (more than 800.000) of orbiting and rotating celestial bodies, perturbation models based on conventional theories of gravitation predict long-term highly unstable states [1, 2] and have a problem with the real stability of the Solar system. Moreover, they do not explain basic facts, for instance, why the Solar system has established the orbital periods 90560 days (Pluto), 60182 (Neptune), 30689 (Uranus), 10759 (Saturn), 4333 (Jupiter), 1682 (Ceres), 687 (Mars), 365 (Earth), 225 (Venus) and 88 days (Mercury). The current distribution of the planetary and lunar orbital and rotational periods appears to them to be completely coincidental.

Recently discovered astrophysical and geophysical cycles of galactic origin suggest that despite the huge number of stars (more than 200 billion), our Galaxy behaves like a precise chronometer as well. Disappointingly, there is no theory of gravitation that derives the correct movement of stars in galaxies or explains at least the existence of galaxies without introducing a huge amount (corrently 68%) of dark energy [3]. In spiral galaxies, the orbiting of stars seems to strongly disobey both Newton's law of universal gravitation and general relativity. Recently, an 85% dark matter [4] universe is required for saving the conventional paradigm.

Perhaps the concept of gravitation itself requires a revision. Obviously, it is not about details, but an important part of the hole is missing.

In this paper we introduce a basic numeric-physical approach that could be the missing link as it allows resolving stability tasks in dynamic systems of any level of complexity.

Methods

In [5] we have shown that the difference between rational, irrational algebraic and transcendental numbers is not only a mathematical task, but it is also an essential aspect of stability in complex dynamic systems. For instance, integer frequency ratios provide resonance interaction that can destabilize a system [6]. Actually, it is transcendental numbers that define the preferred ratios of quantities which avoid destabilizing resonance interaction [7]. In this way, transcendental ratios of quantities sustain the lasting stability of periodic processes in complex dynamic systems. With reference to the evolution of a planetary system and its stability, we may therefore expect that the ratio of any two orbital periods should finally approximate a transcendental number [8].

Among all transcendental numbers, Euler's number $e = 2.71828\ldots$ is unique, because its real power function e^x coincides with its own derivatives. In the consequence, Euler's number allows inhibiting resonance interaction regarding any interacting periodic processes and their derivatives. Because of this unique property of Euler's number, complex dynamic systems tend to establish relations of quantities that coincide with values of the natural exponential function e^x for integer and rational exponents x.

Therefore, we expect that periodic processes in real systems prefer frequency ratios close to Euler's number and its rational powers. Consequently, the logarithms of their frequency ratios should be close to integer $0, \pm 1, \pm 2, \ldots$ or rational values $\pm 1/2, \pm 1/3, \pm 1/4, \ldots$ In [9] we exemplified our hypothesis in particle physics, astrophysics, cosmology, geophysics, biophysics and engineering.

Based on this hypothesis, we introduced a fractal model of matter [10] as a chain system of harmonic quantum oscillators and could show the evidence of this model for all known hadrons, mesons, leptons and bosons as well. In [11] we have shown that the set of stable eigenstates in such systems is fractal and can be described by finite continued fractions:

$$\mathcal{F}_{jk} = \ln\left(\omega_{jk}/\omega_{00}\right) = \langle n_{j0}; n_{j1}, n_{j2}, \ldots, n_{jk} \rangle \qquad (1)$$

where ω_{jk} is the set of angular eigenfrequencies and ω_{00} is the fundamental frequency of the set. The denominators are integer: $n_{j0}, n_{j1}, n_{j2}, \ldots, n_{jk} \in \mathrm{Z}$. The cardinality $j \in \mathrm{N}$ of the set and the number $k \in \mathrm{N}$ of layers are finite. The numeric occupancy of one layer does not influence the numeric occupancy of other layers, so that each layer can be considered as an independent dimension of a k-dimensional manifold. In the canonical form, all numerators equal 1. We use angle brackets for continued fractions.

Any finite continued fraction represents a rational number [12]. Therefore, the ratios ω_{jk}/ω_{00} of eigenfrequencies are always irrational, because for rational exponents the natural exponential function is transcendental [13]. This circumstance provides for lasting stability of those eigenstates of a chain system of harmonic oscillators because it prevents resonance interaction [14] between the elements of the system. The distribution density of stable eigenstates reaches local maxima near reciprocal integers $\pm 1/2, \pm 1/3, \pm 1/4, \ldots$ that are attractor points (fig. 1) in the fractal set $\mathcal{F}_{jk}$ of natural logarithms. Integer logarithms $0, \pm 1, \pm 2, \ldots$ represent the most stable eigenstates (main attractors).

In the case of harmonic quantum oscillators, the continued fractions $\mathcal{F}_{jk}$ define not only fractal sets of natural angular frequencies ω_{jk}, angular accelerations $a_{jk} = c \cdot \omega_{jk}$, oscillation periods $\tau_{jk} = 1/\omega_{jk}$ and wavelengths $\lambda_{jk} = c/\omega_{jk}$ of the chain system, but also fractal sets of energies $\mathrm{E}_{jk} = \hbar \cdot \omega_{jk}$ and masses $\mathrm{m}_{jk} = \mathrm{E}_{jk}/c^2$ which correspond with the eigenstates of the system. For this reason, we call the continued fraction $\mathcal{F}_{jk}$ the *Fundamental Fractal* of stable eigenstates in chain systems of harmonic quantum oscillators.

Fig. 1: The distribution of stable eigenvalues of $\mathcal{F}_{jk}$ for $k = 1$ (above) and for $k = 2$ (below) in the range $-1 \leqslant \mathcal{F}_{jk} \leqslant 1$.

In fact, scale relations in particle- [10] and astrophysics [15] obey the same Fundamental Fractal (1), without any additional or particular settings. The proton-to-electron rest energy ratio approximates the first layer of the Fundamental Fractal that could explain their exceptional stability. In fact, the life-spans of the proton and electron top everything that is measurable, exceeding 10^{29} years [16].

Property	Electron	Proton
$\mathrm{E} = \mathrm{mc}^2$	0.5109989461(31) MeV	938.2720813(58) MeV
$\omega = \mathrm{E}/\hbar$	$7.76344 \cdot 10^{20}$ Hz	$1.42549 \cdot 10^{24}$ Hz
$\tau = 1/\omega$	$1.28809 \cdot 10^{-21}$ s	$7.01515 \cdot 10^{-25}$ s
$\lambda = c/\omega$	$3.86159 \cdot 10^{-13}$ m	$2.10309 \cdot 10^{-16}$ m

Table 1: The basic set of the physical properties of the electron and proton. Data from Particle Data Group [16]. Frequencies, oscillation periods and wavelengths are calculated.

The proton-to-electron ratio (tab. 1) approximates the seventh power of Euler's number and its square root:

$$\ln\left(\frac{\omega_p}{\omega_e}\right) = \ln\left(\frac{1.42549 \cdot 10^{24}\ \mathrm{Hz}}{7.76344 \cdot 10^{20}\ \mathrm{Hz}}\right) \simeq 7 + \frac{1}{2} = \langle 7; 2 \rangle$$

In the consequence of this potential difference of the proton relative to the electron, the scaling factor $\sqrt{e} = 1.64872\ldots$ connects attractors of proton stability with similar attractors of electron stability in alternating sequence.

These unique properties of the electron and proton predestinate their physical characteristics as fundamental units. Table 1 shows the basic set of electron and proton units that can be considered as a fundamental metrology (c is the speed of light in a vacuum, $\hbar$ is the Planck constant). In [11] was shown that the fundamental metrology (tab. 1) is completely compatible with Planck units [17]. Originally proposed in 1899 by Max Planck, these units are also known as natural units, because the origin of their definition comes only from properties of nature and not from any human construct. Max Planck wrote [18] that these units, "regardless of any particular bodies or substances, retain their importance for all times and for all cultures, including alien and non-human, and can therefore be called natural units of measurement". Planck units reflect the characteristics of space-time.

We assume that scale invariance according to the Fundamental Fractal (1), which is calibrated to the physical properties of the proton and the electron, is a universal characteristic of organized matter and criterion of stability. This hypothesis we have called *Global Scaling* [9].

In [19] we applied the Fundamental Fractal (1) to macroscopic scales interpreting gravity as quantum attractor effect of its stable eigenstates. We have shown that the orbital and rotational periods of planets, planetoids and large moons of the solar system correspond with attractors of electron and proton stability [11]. This is valid also for exoplanets [15] of the systems Trappist 1 and Kepler 20. In [8] we have shown that the maxima in the frequency distribution of the orbital periods of 1430 exoplanets listed in [20] correspond with attractors of the Fundamental Fractal. In [21] we have shown that the maxima in the frequency distribution of the number of stars in the solar neighborhood as function of the distance between them correspond well with attractors of the Fundamental Fractal.

In this paper we will show that the Fundamental Fractal (1) determines also the Earth axial precession cycle, the obliquity variation cycle as well as the apsidal precession cycle and the orbital eccentricity cycle. In addition, we will show that recently discovered geological cycles, as well as the periodic variations in the movement of the Solar system through the Galaxy, substantiate their determination by the Fundamental Fractal.

Results

Since its birth the Sun has made about 20 cycles around the Galaxy, and during this time the Solar system has made many passages through the spiral arms of the disk. The Sun's orbit in the Galaxy is not circular. There are temporal variations in the distance from the Galactic center with a period of T_S = 170 million years [22] that corresponds precisely with the main attractor $\langle 90 \rangle$ of proton stability of the Fundamental Fractal (1):

$$\ln\left(\frac{T_S}{2\pi \cdot \tau_p}\right) = 90$$

$2\pi \cdot \tau_p$ is the oscillation period of the proton (tab. 1). The recently [23] discovered geological cycle with a period of

$T_G = 27$ million years corresponds well with the same attractor ⟨90⟩, but relative to the angular oscillation period of the proton:

$$\ln\left(\frac{T_G}{\tau_p}\right) = 90$$

The connection $T_S = 2\pi \cdot T_G$ suggests that the 27 million years' geological cycle could be caused by angular components of the periodical variations of the distance of the Solar system (and the Earth) from the Galactic center. In addition, [23] reports a geological cycle of 8.9 Ma that approximates the main attractor ⟨87⟩ of proton stability:

$$\ln\left(\frac{8.9\ Ma}{2\pi \cdot \tau_p}\right) = 87$$

The Sun's path oscillates above and below the Galactic plane with a period of approximately 63 million years [22] that coincides with the main attractor ⟨89⟩ of proton stability:

$$\ln\left(\frac{63\ Ma}{2\pi \cdot \tau_p}\right) = 89$$

Earth's axial precession cycle (25,770 years) fits the attractor ⟨83⟩ of proton stability:

$$\ln\left(\frac{25,770\ a}{\tau_p}\right) = 83$$

By the way, 25,770 years is also the time it takes for a signal to travel from the Galactic center to Earth at the speed of light.

The Fundamental Fractal (1) is of pure numeric origin, and there is no particular physical mechanism that creates it. It is all about transcendental ratios of frequencies [8] that inhibit destabilizing resonance interaction. In this way, the Fundamental Fractal concerns all repetitive processes, independently on their temporal or spatial scales.

For instance, Earth's apsidal precession cycle and orbital eccentricity cycle (both of 112,000 years) correspond with the attractor ⟨77⟩ of electron stability:

$$\ln\left(\frac{112,000\ a}{\tau_e}\right) = 77$$

τ_e is the angular oscillation period of the electron (tab. 1). Earth's obliquity variation cycle (41,000 years) corresponds with the attractor ⟨76⟩ of electron stability:

$$\ln\left(\frac{41,000\ a}{\tau_e}\right) = 76$$

Naturally, we expect the existence of further galactic cycles that correspond with other main attractors of the Fundamental Fractal. Table 2 gives an overview of expected main attractor cycles in the scale of millions of years.

n	$T_p(n)$, Ma	$t_p(n)$, Ma	n	$T_e(n)$, Ma	$t_e(n)$, Ma
91	463.35	73.75	83	285.41	45.42
90	170.46	27.13	82	105.00	16.71
89	62.71	9.98	81	38.62	6.15
88	23.07	3.67	80	14.21	2.26
87	8.49	1.35	79	5.23	0.83
86	3.12	0.50	78	1.92	0.31

Table 2: Cycles corresponding with main attractors of proton and electron stability in the range of millions of years (Ma).

Every attractor of proton or electron stability defines the period of a stable cycle and its angular period. As main attractors correspond with integer exponents n of the Fundamental Fractal (1), it is easy to calculate main attractor cycles:

$$t_e(n) = \tau_e \cdot e^n \qquad T_e(n) = 2\pi \cdot t_e(n)$$

$$t_p(n) = \tau_p \cdot e^n \qquad T_p(n) = 2\pi \cdot t_p(n)$$

In general, the identification of the predicted galactic cycles requires a significant increase in current data precision.

Conclusion

Within our approach, numeric attractors of stability determine the distribution of matter in space and time. Since the distribution of the attractors is fractal, the distribution of matter is also fractal. Numerical attractors cause effects known as gravity, electricity, magnetism, and nuclear forces. Numerical relationships are primary, physical effects are secondary. Numerical attractors cause the formation of matter in all scales – from the electron and proton up to planets, stars and galaxies. Interscalar cosmology [9] bases on this approach.

In particular, for maintaining stability of motion, the Sun does not have to avoid parametric resonance with every single other star on its path through the Galaxy. As this task cannot be resolved in general, the application of transcendental frequency ratios appears to be a significant alternative. As we have shown, not only stars [21], but also planets [8] make extensive use of it.

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